PRACTICE PROBLEM SET 3

DUE DATE: -

- **Sections 3.5 - 4.3**
- Questions are either directly from the text or a small variation of a problem in the text.
- The terms in the bracket indicate the problem number from the text.

Section 3.5

1) (Switching the order of differentiation and integration for the inhomogeneous solution to the heat equation) Consider the inhomogeneous solution to the heat equation given by

$$u(x,t) = \int_0^t \int_{-\infty}^{\infty} S(x-y, t-s)\, f(y,s)\; dy\, ds\,.$$

If f is continuous, bounded and satisfies,

$$\int_{-\infty}^{\infty} |f(y,t)|\, dy \le M \quad \forall t \ge 0\,,$$

Prove that

$$\partial_x u(x,t) = \int_0^t \int_{-\infty}^{\infty} \partial_x S(x-y, t-s)\, f(y,s)\; dy\, ds\,.$$

Section 4.1

2) Complete exercise 8.1 in the lecture notes.

3) (Prob 3, Pg 89) A quantum-mechanical particle on the line with an infinite potential outside, satisfies the Schrodinger equationn equation with Dirichlet conditions at both ends. Separate the variables and find a representation for the solution.

$$\begin{aligned} u_t &= i u_{xx} \quad 0 < x < \ell \\ u(0,t) &= u(\ell,t) = 0 \\ u(x,0) &= \phi(x) \end{aligned}$$

4) (Prob 6, Pg 89) Separate the variables for the equation

$$t u_t = u_{xx} + 2u\,,$$

with the bounddary conditions

$$u(0,t) = u(\pi,t) = 0\,.$$

Show that there an infinite number of solutions that satisfy the initial condition $u(x,0) = 0$. Is the problem well-posed?

Section 4.2

5) (Prob 1, Pg 92) Solve the diffusion problem with mixed boundary conditions:

$$\begin{aligned} u_t &= k u_{xx} \quad 0 < x < \ell \\ u(0,t) &= u_x(\ell,t) = 0 \\ u(x,0) &= \phi(x) \end{aligned}$$

Section 4.3

6) (Prob 7, Pg 100) Consider the Robin eigenvalue value problem

$$X'' = -\lambda X$$
$$X'(0) - aX(0) = X'(\ell) + aX(\ell) = 0\,.$$

Show that as $a \to +\infty$, the eigenvalues tend to the eigenvalues of the Dirichlet problem, i.e. if $\beta_n(a)^2$ is the $(n+1)$st eigenalue, then

$$\lim_{a\to\infty}\left\{\beta_n(a) - \frac{(n+1)\pi}{\ell}\right\} = 0\,.$$