# PROBLEM SET 5

DUE DATE: - APR 11

- **Chap 5**
- Questions are either directly from the text or a small variation of a problem in the text.
- Collaboration is okay, but final submission must be written individually. Mention all collaborators on your submission.
- The terms in the bracket indicate the problem number from the text.

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**Section 6.1**

**1)** (Prob 7, Pg 160) Solve $u_{xx} + u_{yy} + u_{zz} = 1$ in the spherical shell $1 < r < 2$, with $u(1, \theta, \phi) = u(2, \theta, \phi) = 0$ for all $\theta, \phi$.

**2)** (Prob 13, Pg 160) A function $u$ is subharmonic in $D$ if it satisfies $\Delta u \geq 0$ in $D$. Prove that it's maximum value is attained on the boundary. Note that the same is not true for the minimum value.

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**Section 6.2**

**3)** (Prob 1, Pg 164) Solve $u_{xx} + u_{yy} = 0$ in the rectangle $0 < x < 1$, $0 < y < 2$ with the following boundary conditions:

$$\begin{aligned} u_x &= -1 \quad x = 0 \\ u_y &= 2 \quad y = 0 \\ u_x &= 0 \quad x = 1 \\ u_y &= 0 \quad y = 2\,. \end{aligned}$$

**4)** (Prob 7, Pg 165) Find the harmonic function in the semi-infinite strip $\{0 \leq x \leq \pi\,, 0 \leq y < \infty\}$ that satisfy the boundary conditions:

$$u(0, y) = u(\pi, y) = 0\,, \quad u(x, 0) = h(x)\,, \quad \lim_{y \to \infty} u(x, y) = 0\,.$$

b) What would be the issue if the condition at $\infty$ is not imposed?

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**Section 6.3**

**5)** (Prob 2, Pg 172) Solve $u_{xx} + u_{yy} = 0$ in the disk $\{r < a\}$ with the boundary condition

$$u(a, \theta) = 1 + 3\sin(\theta)\,.$$

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**Section 6.4**

**6)** (Prob 1, Pg 175) Solve $u_{xx} + u_{yy} = 0$ in the exterior $\{r > a\}$ of the disk, with the boundary condition $u(a, \theta) = 1 + 3\sin(\theta)$ and the condition that $u$ remains bounded as $r \to \infty$

**7)** (Prob 4, Pg 176) Derive Poisson's formula for the exterior of a circle.