PROBLEM SET 5

DUE DATE: APR 11

- Chap 5
- Questions are either directly from the text or a small variation of a problem in the text.
- Collaboration is okay, but final submission must be written individually. Mention all collaborators on your submission.
- The terms in the bracket indicate the problem number from the text.

Section 6.1
1) (Prob 7, Pg 160) Solve $u_{xx} + u_{yy} + u_{zz} = 1$ in the spherical shell $1 < r < 2$, with $u(1, \theta, \phi) = u(2, \theta, \phi) = 0$ for all $\theta, \phi$.
2) (Prob 13, Pg 160) A function $u$ is subharmonic in $D$ if it satisfies $\Delta u \geq 0$ in $D$. Prove that it’s maximum value is attained on the boundary. Note that the same is not true for the minimum value.

Section 6.2
3) (Prob 1, Pg 164) Solve $u_{xx} + u_{yy} = 0$ in the rectangle $0 < x < 1, 0 < y < 2$ with the following boundary conditions:
   \begin{align*}
u_x &= -1 & x &= 0 \\
u_y &= 2 & y &= 0 \\
u_x &= 0 & x &= 1 \\
u_y &= 0 & y &= 2.
\end{align*}
4) (Prob 7, Pg 165) Find the harmonic function in the semi-infinite strip \{0 \leq x \leq \pi, 0 \leq y < \infty\} that satisfy the boundary conditions:
   \begin{align*}
u(0, y) &= u(\pi, y) = 0, & \lim_{y \to \infty} u(x, y) &= 0.
\end{align*}

b) What would be the issue if the condition at $\infty$ is not imposed?

Section 6.3
5) (Prob 2, Pg 172) Solve $u_{xx} + u_{yy} = 0$ in the disk \{r < a\} with the boundary condition
   \begin{align*}
u(a, \theta) &= 1 + 3 \sin(\theta).
\end{align*}

Section 6.4
6) (Prob 1, Pg 175) Solve $u_{xx} + u_{yy} = 0$ in the exterior \{r > a\} of the disk, with the boundary condition $u(a, \theta) = 1 + 3 \sin(\theta)$ and the condition that $u$ remains bounded as $r \to \infty$.
7) (Prob 4, Pg 176) Derive Poisson’s formula for the exterior of a circle.