PROBLEM SET 4

DUE DATE: - MAR 28

- **Chap 5**
- Questions are either directly from the text or a small variation of a problem in the text.
- Collaboration is okay, but final submission must be written individually. Mention all collaborators on your submission.
- The terms in the bracket indicate the problem number from the text.

Section 5.1

1) (Prob 2, Pg 111) Compute the fourier sine and fourier cosine series of $\phi(x) = x^2$ for $0 \leq x \leq 1$.

2) (Prob 10, Pg 111) A string (of tension T and density ρ) with fixed ends at $x = 0$ and $x = \ell$ is hit by a hammer so that $u(x, 0) = 0$ and $\partial_t u(x, 0) = V$ for $x \in \left[-\delta + \frac{1}{2}\ell, \delta + \frac{1}{2}\ell\right]$ and $\partial_t u(x, 0) = 0$ otherwise. Find the solution explicitly in series form. Find the energy

$$E_n[h](t) = \frac{1}{2}\int_0^\ell \left[\rho \partial_t h(x,t)^2 + T\partial_x h(x,t)^2\right] dx$$

of the nth harmonic $h = h_n$. Conclude that if δ is small (a concentrated blow), each of the first few overtones has almost as much energy as the fundamental mode.

Section 5.3

3) (Prob 4, Pg 123) Consider the problem $u_t = ku_{xx}$ for $0 < x < \ell$, with the boundary conditions $u(0, t) = U$, $u_x(\ell, t) = 0$, and the initial condition $u(x, 0) = 0$, where U is a constant.

a) Find the solution in series form (Consider $u(x, t) - U$)

b) Using a direct argument, show that the series converges for each time slice $t = t_0$ uniformly in x for $t_0 > 0$.

4) (Prob 9, Pg 123) Show that the boundary conditions

$$X(b) = \alpha X(a) + \beta X'(a) \quad \text{and} \quad X'(b) = \gamma X(a) + \delta X'(a)$$

on an interval $a \leq x \leq b$ are symmetric if and only if $\alpha\delta - \beta\gamma = 1$.

Section 5.4

5) (Prob 1, Pg 134) $\sum_{n=0}^{\infty} (-1)^n x^{2n}$ is a geometric series.

a) Does it converge pointwise in the interval $-1 < x < 1$?

b) Does it converge uniformly in the interval $-1 < x < 1$?

c) Does it converge in the $\mathbb{L}^2$ sense in the interval $-1 < x < 1$?

6) (Prob 3, Pg 134) Let $f_n(x)$ be the sequence of functions defined as follows: $f_n(0.5) = 0$, $f_n(x) = \gamma_n$ in the interval $[0.5 - \frac{1}{n}, 0.5)$, let $f_n(x) = -\gamma_n$ in the interval $(0.5, 0.5 + \frac{1}{n}]$ and let $f_n(x) = 0$ elsewhere. Show that:

a) $f_n(x) \to 0$ pointwise.

b) The convergence is not uniform.

c) $f_n \to 0$ in the $\mathbb{L}^2$ sense if $\gamma_n = n^{1/3}$

d) f_n does not converge in the L^2 sense if $\gamma_n = n$.