• Chap 5
• Questions are either directly from the text or a small variation of a problem in the text.
• Collaboration is okay, but final submission must be written individually. Mention all collaborators on your submission.
• The terms in the bracket indicate the problem number from the text.

Section 5.1
1) (Prob 2, Pg 111) Compute the fourier sine and fourier cosine series of \( \phi(x) = x^2 \) for \( 0 \leq x \leq 1 \).
2) (Prob 10, Pg 111) A string (of tension \( T \) and density \( \rho \)) with fixed ends at \( x = 0 \) and \( x = \ell \) is hit by a hammer so that \( u(x, 0) = 0 \) and \( \partial_t u(x, 0) = V \) for \( x \in [-\delta + \frac{\ell}{2}, \delta + \frac{\ell}{2}] \) and \( \partial_t u(x, 0) = 0 \) otherwise. Find the solution explicitly in series form. Find the energy \( E_n[h](t) = \frac{1}{2} \int_0^\ell \left[ \rho \partial_t h(x, t)^2 + T \partial_x h(x, t)^2 \right] dx \) of the \( n \)th harmonic \( h = h_n \). Conclude that if \( \delta \) is small (a concentrated blow), each of the first few overtones has almost as much energy as the fundamental mode.

Section 5.3
3) (Prob 4, Pg 123) Consider the problem \( u_t = ku_{xx} \) for \( 0 < x < \ell \), with the boundary conditions \( u(0, t) = U, u_x(\ell, t) = 0 \), and the initial condition \( u(x, 0) = 0 \), where \( U \) is a constant.
   a) Find the solution in series form (Consider \( u(x, t) - U \))
b) Using a direct argument, show that the series converges for each time slice \( t = t_0 \) uniformly in \( x \) for \( t_0 > 0 \).
4) (Prob 9, Pg 123) Show that the boundary conditions
   \[ X(b) = \alpha X(a) + \beta X'(a) \quad \text{and} \quad X'(b) = \gamma X(a) + \delta X'(a) \]
on an interval \( a \leq x \leq b \) are symmetric if and only if \( \alpha \delta - \beta \gamma = 1 \).

Section 5.4
5) (Prob 1, Pg 134) \( \sum_{n=0}^{\infty} (-1)^n x^{2n} \) is a geometric series.
   a) Does it converge pointwise in the interval \(-1 < x < 1\)?
b) Does it converge uniformly in the interval \(-1 < x < 1\)?
c) Does it converge in the \( L^2 \) sense in the interval \(-1 < x < 1\)?
6) (Prob 3, Pg 134) Let \( f_n(x) \) be the sequence of functions defined as follows: \( f_n(0.5) = 0, f_n(x) = \gamma_n \) in the interval \([0.5 - \frac{1}{n}, 0.5]\), let \( f_n(x) = -\gamma_n \) in the interval \((0.5, 0.5 + \frac{1}{n}] \) and let \( f_n(x) = 0 \) elsewhere. Show that:
   a) \( f_n(x) \rightarrow 0 \) pointwise.
b) The convergence is not uniform.
c) \( f_n \rightarrow 0 \) in the \( L^2 \) sense if \( \gamma_n = \pi^{1/3} \)
d) \( f_n \) does not converge in the \( L^2 \) sense if \( \gamma_n = n \).