1. Suppose that $R$ and $L$ are the standard right shift and left shift operators on $\ell^2(\mathbb{N})$. Show that
\[
\sigma_p(R) = \emptyset, \quad \sigma_c(R) = \{ \lambda \in \mathbb{C} : |\lambda| = 1 \}, \quad \sigma_r(R) = \{ \lambda \in \mathbb{C} : |\lambda| < 1 \}, \quad \sigma_p(L) = \{ \lambda \in \mathbb{C} : |\lambda| < 1 \}, \quad \sigma_c(L) = \{ \lambda \in \mathbb{C} : |\lambda| = 1 \}, \quad \sigma_r(L) = \emptyset.
\]

2. Suppose that $e_i$ are the standard coordinate vectors in $\ell^2(\mathbb{N})$ and $T$ is the operator defined by
\[
T e_i = \lambda_i e_i,
\]
where $\lambda_i$ are distinct. Show that $\sigma_r(T) = \emptyset$ and that
\[
\lambda \in \{ \lambda_i \} \setminus \{ \lambda_i \} \in \sigma_c(T)
\]

3. Construct a compact operator $T$ such that $0$ is in a) the point spectrum, b) the continuous spectrum, and c) the residual spectrum.

4. Suppose that $\lambda \in \mathbb{R}$. Show that if $\lambda \in \sigma_p(T)$ and $\lambda \notin \sigma_p(T^*)$, then $\lambda \in \sigma_r(T^*)$. 