Problem set 7

Due date: Apr 23

April 16, 2018

1. Suppose that R and L are the standard right shift and left shift operators on $\ell^2(\mathbb{N})$. Show that

$$\sigma_p(R) = \emptyset\,, \quad \sigma_c(R) = \{\lambda \in \mathbb{C} : |\lambda| = 1\}\,, \quad \sigma_r(R) = \{\lambda \in \mathbb{C} : |\lambda| < 1\}\,,$$

$$sigma_p(L) = \{\lambda \in \mathbb{C} : |\lambda| < 1\}\,, \quad \sigma_c(L) = \{\lambda \in \mathbb{C} : |\lambda| = 1\}\,, \quad sigma_r(L) = \emptyset\,.$$

2. Suppose that e_i are the standard coordinate vectors in $\ell^2(\mathbb{N})$ and T is the operator defined by

$$Te_i = \lambda_i e_i\,,$$

where λ_i are distinct. Show that $\sigma_r(T) = \emptyset$ and that

$$\lambda \in \overline{\{\lambda_i\}} \setminus \{\lambda_i\} \in \sigma_c(T)$$

3. Construct a compact operator T such that 0 is in a) the point spectrum, b) the continuous spectrum, and c) the residual spectrum.

4. Suppose that $\lambda \in \mathbb{R}$. Show that if $\lambda \in \sigma_p(T)$ annd $\lambda \notin \sigma_p(T^*)$, then $\lambda \in \sigma_r(T^*)$.