SPECIAL RELEASE ON MANDELBROT'S CONTRIBUTIONS TO PHYSICS

NEW HAVEN, CONN. Benoit Mandelbrot, Sterling Professor of Mathematical Sciences at Yale University and the "father of fractals," shared the 2003 Japan Prize for Science and Technology

Through numerous contributions of fractal geometry to physics, Mandelbrot can arguably be called the originator of a first quantitative approach to the study of roughness. As background, keep in mind that acoustics became quantitative when "pure sounds" were defined as being periodic, making pitch measurable by a frequency. As had to be the case, this quantitative measure is consistent with "intuition" and earlier knowledge as manifested for example, in music. Similarly, the theory of heat became quantitative when Galileo devised the thermometer and measured hotness by a temperature;

Most interestingly, and despite the fact that much of nature in the raw can be called rough, a quantitative measure of roughness was not available until fractal geometry was developed. Mandelbrot layed the foundation for a future “rugometry” when he discovered [FGN, 103] that ubiquitous examples of rough curves or surfaces are self-similar or self-affine, and as a first measure of such "pure" roughness he proposed fractal dimension or codimension. Those concepts had arisen in esoterica, as the Hausdorff-Besicovitch dimension and Holder-Lipschitz exponent. It was therefore necessary to first reinterpret them as being numerical characteristics of an invariance, and then expand their study, both concretely and intuitively. Rough surfaces may also satisfy weaker forms of invariance. For them, measurement may require one of Mandelbrot's mathematical inventions, namely, multifractals.

Like pure sound or pure elliptic motion under gravitation, pure roughness is an abstraction but a useful one. For example, experiment shows that metal fractures are fractal over the very broad range of sizes covering five decades at least. The range is sometimes even broader, but may be limited by the nature of the data.

In topics that belong to core physics, Mandelbrot concentrated his attention on the numerous specific topics in which roughness is the key unsolved ingredient. A master in the use of the fractal tools that he identified or designed to handle roughness, he extended the range of topics he could handle by collaborating with eminent “insiders”. The topics he tackled range over fluid mechanics, disordered systems, anomalous fluctuations, the large scale structure of the universe, and non-linear dynamics. His studies clarified many old issues, opened many questions, and provoked extensive work.

In the context of turbulence, Mandelbrot conjectured [FGN, Chapter 10] that the shapes it creates are fractal and [72] that its intermittency is modeled by a multifractal measure, a tool that he specifically developed for this purpose. Both conjectures have been confirmed experimentally by K. Sreenivasan and multifractals became widely used in many other areas of physics.

In the field of critical phenomena, Mandelbrot conjectured that infinitely downsized percolation clusters converge towards fractal curves. This mathematical conjecture, since proven rigorously by S. Smirnov, underpins a series of publications with A. Aharony, Y. Gefen and others [90, 93, 96, 100, 101, 102, 105, 110]. Those publications established, and this is now universally accepted, that the clusters themselves, as well as portions of clusters that play diverse specific roles, should be viewed as fractals. This added a fresh example to the existing cases where it can be said that physics is ruled by geometry.

Diffusion limited aggregates (DLA) are a new, fascinating and difficult form of clusters. L. Sander, who discovered them with T. Witten, was quoted as pointing out that “numerous researchers had come close to discovering DLA, but didn't know the implications of what they had. I attribute [our insight] to Mandelbrot's [urging us] to think about odd shapes". “It was amazing", adds Witten", to hit on something that captured peoples' imagination even a tenth as much as this, and for it to keep going. But what's truly unexpected is that this problem just resists being solved".

The comments in one or both last

sentences are also made about many other aspects of fractals. In the meantime, almost everything that is known about DLA was obtained using fractal tools, and the problem's continuing difficulty does not repel Mandelbrot but attracts him. He has worked extensively on the "deviations" of DLA from exact self-similarity [127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 148, 153, 157, 179].

Another long-range issue is that of anomalous fluctuations discussed in many chapters of Mandelbrot’s books on “Multifractals and 1/f Noise” and “Gaussian Self-Affinity and Fractals.” Because many noises share a "1/f" spectrum, they became interpreted as different instances of a single unified physical phenomenon for which a single explanation could be achieved -- albeit with no success so far. To the contrary, Mandelbrot showed that 1/f noises are examples of roughness but can be highly unlike one another. Therefore traditional tools such as spectra do represent them quantitatively, but only partially, and miss their most significant features. Using fractal tools, Mandelbrot identified at least three sharply distinct kinds of 1/f noise he now calls “unifractal”, “mesofractal” and “multifractal”. Each category involves a different geometry and demands its own sort of fractal tools.

J. Peebles and his school were taking for granted that the large scale structure of the universe is homogeneous, unevenness being restricted to a local scale extending to a crossover at 5 megaparces. They observed that a 1/f spectrum prevails “locally,” up to 5 megaparsecs, but did not seek a geometric description. Mandelbrot provided one [FGN Chapter 10]. His first step consisted in observing that if homogeneity is not assumed in advance, a correct statistical analysis moves the crossover from 5 megaparces to a much larger value. A second step consisted in reinterpreting those spectra and other evidence as symptoms of an underlying fractal structure extending well beyond 5 megaparces. A very careful analysis by L. Pietronero convinced a wide community to move the crossover to at least 200-300 megaparces. (Even higher values are quoted but continue to be controversial.)

A third step taken by Mandelbrot faced the criticism that clustering must introduce a “center of the universe” or postulate a hierarchy. He showed that neither fatal blemish is present in two otherwise cartoonish fractal “scenarios”. Both are very easy to simulate and either suffices to account simultaneously for the observed spectra and also for the observed clusters, voids, and filaments. Those scenarios’ ability to mimic the actual maps challenges many traditional beliefs. It raises the question of whether those geometric structures are physically real or, to some unspecified extent, artifacts of the human mind. Their effectiveness suggests the existence of other contexts where seemingly separate experimental findings simply manifest an underlying fractality.

In the study of interstellar gas clouds, fractality also proved essential and encountered no controversy.

In the context of dynamics, Mandelbrot's now classic study brought the non-linear map z x z + c back from the real line to the complex plane where it had started with Fatou and Julia. This led him to discover and describe the object now called Mandelbrot set M, which is discussed in the Special Release on pure mathematics. In physics, it has become the “icon” of the contrast between regular and chaotic behavior. It has also been called “the most complex object in mathematics”. Many facts about it remain unsolved, such as Mandelbrot's main conjecture that M is the closure of M°, defined as the set of values of c in parametric space for which there exists a finite stable limit cycle.

Engineering is affected as well as inventors keep expanding the uses of fractals. Most recently studied are fractal antennas, fractal capacitors that fit a farad on a pinhead, and fractal combustion chambers in chemical engineering.

Through dynamics, the Mandelbrot set made mathematics and physics attractive to millions of students. It is not often in human history that a mathematical inspiration relative to basic physical problems penetrates so far into the lives of ordinary people, changes their vision of the world, and spawns new sorts of industries and arts.