SPECIAL RELEASE ON MANDELBROT'S CONTRIBUTIONS TO PURE MATHEMATICS

NEW HAVEN, CONN. Benoit Mandelbrot, Sterling Professor of Mathematical Sciences at Yale University and the "father of fractals," shared the 2003 Japan Prize for Science and Technology.

Mandelbrot's contributions to pure mathematics led, on the one hand, to the reinterpretation of existing mathematics and, on the other hand, to the creation of new mathematics. Most derive from the use of computer graphics, of which he was a pioneer and remains a master.

The telling pictures he drew of old standbys like the Koch or Peano curves and the Cantor set thoroughly disproved the previously held belief that those sets are "monsters." Quite to the contrary, he turned them around into unavoidable "cartoons" of a reality that science had felt powerless to tackle, namely, the overwhelming fact that most of raw nature is not smooth but very rough. For example, he reinterpreted Peano "curves" as nothing but motions following a plane-filling network of rivers. More profoundly, Mandelbrot destroyed the belief, near-universal among pure mathematicians around 1980, that a picture can only lead to another, and never to fresh mathematics. Near single-handedly at first, he provided a variety of examples of pictures that were originally meant to respond to the needs of a science but have in addition triggered purely mathematical conjectures.

Altogether, he is not known mainly for providing difficult proofs but for many separate mathematical conjectures of all kinds, each of which seems to have opened a new field that continues and prospers long after he has moved to other concerns. Norbert Wiener once described his key contribution to science as bringing together - starting from widely opposite horizons -the fine mathematical points of Lebesgue integration and the physics of Gibbs and Perrin. Also (like Poincaré) Wiener was very committed (and successful) in making frontier science known to a wide public. On both accounts it is easy to argue that the closest and most brilliant living heir of Wiener's is Benoit Mandelbrot. His theory of fractals is arguably a multiple second flowering of Wiener's Brownian motion.

Here is a list of a few of Mandelbrot's contributions to pure mathematics. They are not ordered chronologically but by decreasing "notoriety."

1. Discovery of the Mandelbrot set M, as reported in 1980 in [91] and elaborated upon in [FGN, Chapter 19, 98, 107, 109, 112]. The classic reference [91] included a multiplicity of observations-conjectures that concern the map from z to $z \times z + c$. They created the field of modern complex dynamics, known to all mathematicians. One of his conjectures involved a form of self-similarity: M includes "islands" close in form to the whole. Another conjecture was that the boundary of M is of Hausdorff dimension 2. These conjectures have been resolved. But the conjecture now called MLC: "the Mandelbrot set is locally connected," is still unresolved. In 1980 Mandelbrot had made the equivalent conjecture that M is the closure of the set M° of c such that $z \times z + c$ has a finite stable limit point. The notion that M° is an interesting object is natural, but its study proved very difficult. The motivation that led Mandelbrot to study M was the conjecture that it is the closure of M°.

Incidentally, most "ordinary" people seem to have heard of the Mandelbrot set: it is arguably the only tangible proof known to them that mathematics is alive and well.

2.   2. Discovery of an algorithm for limit sets of certain Kleinian groups [95]. The search for such an algorithm started with H. Poincaré and F. Klein and has been going on for a hundred years. Its discovery has sparked wide work exemplified by a recent book by D. Mumford et al.

3.   3. Introduction of the concepts of Brownian cluster and Brownian boundary in 1982 [TFGN, page 243.] This was accompanied by the conjecture that the boundary's Hausdorff dimension is 4/3. Combined with related conjectured dimensions for percolation and Ising clusters, the diverse 4/3 grew into sharp challenges to the analysts and led since 1998 to widely

acclaimed proofs by Duplantier, Lawler, Schramm, Werner, and Smirnov. Earlier, a dozen or so scattered technical conjectures in analysis had been shown to be equivalent to that "4/3." Therefore, all have now been proven as corollaries and together provide an element of unity that continues to be explored.

4.   4. Random multiplicative singular measures constructed from 1968 to 1974 and now called

multifractal. They were not intended as new esoterica but a model of turbulence and finance. The conjectures Mandelbrot put forward in [55, 56, 72] and many later papers [82, 116, 120, 125, 126, 132, 140, 152, 180, 183, 184, 185] created an active and prosperous subbranch of mathematics and the main current branch of statistical modeling of the variation of financial prices. From microcanonical products to canonical ones and recently to products of pulses [180] and of other kinds of functions [183], the constraints on the construction were made less and less strong and immensely richer structures arose.

1.   5. Conjecture in 1975 [TFGN, chapter 11] that the solution of partial differential equations become fractal in due time. This conjecture continues to inspire work along many lines. They propose to justify (if not yet to explain) turbulences as a singularity of Navier-Stokes equations, and the fractal distribution of galaxies as created by the Laplace equation with non fixed irregularities.

2.   6. Conjecture in 1980, [90], that Laplacian diffusion on fractals is overwhelmingly influenced by what used to be an esoteric topological property of a fractal: its Menger Urysohn "order of ramification." In fact, finite, resp. infinite, ramification demands very different approaches. An active school examines these topics, exemplified by a book by Kigami.

3.   7. The study of the connectedness of random fractals led in [TFGN p. 216] to several conjectures on percolation in fractal clusters. They were proved to great acclaim by Chayes, Chayes, and Durrett, then generalized and shown to exhibit a stunning variety of odd behaviors.

4.   8. Further mathematical observations and conjectures beyond counting, relative to many other topics, are scattered through TFGN and the first three volumes of Mandelbrot's *Selected Papers*. These may deserve a careful look.