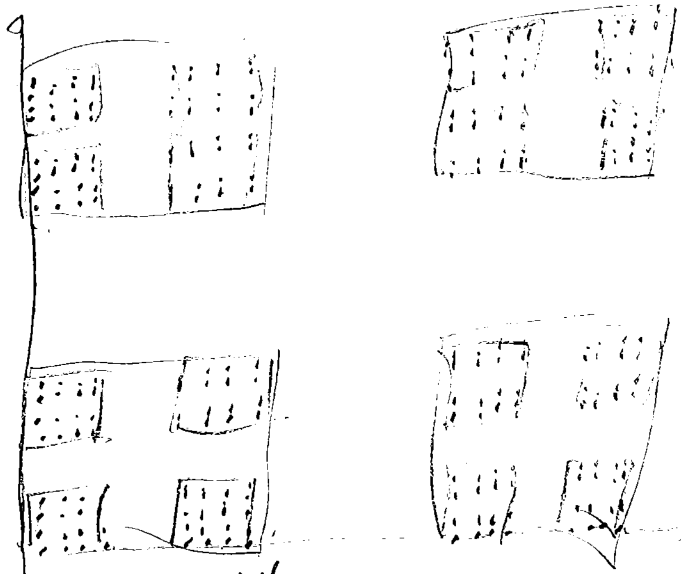


$N(\frac{1}{3}) = 4 \qquad N(\frac{1}{9}) = 16 \qquad N(\frac{1}{27}) = 64$

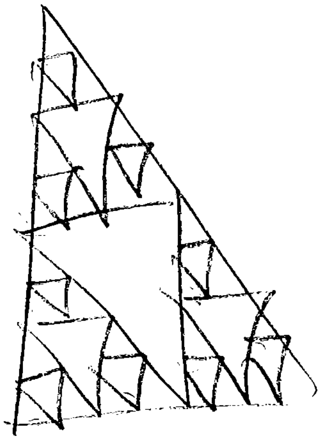
general pattern: $N(\frac{1}{3^n}) = 4^n$

$$d = \lim_{n\to\infty} \frac{\log(N(1/3^n))}{\log(1/(1/3^n))} = \lim_{n\to\infty} \frac{\log(4^n)}{\log(3^n)}$$

$$= \lim_{n\to\infty} \frac{\cancel{n}\log(4)}{\cancel{n}\log(3)} = \frac{\log(4)}{\log(3)}$$

Product rule for dimensions:

$$\dim(A\times B) = \dim(A) + \dim(B)$$



gasket $\cup$ line segment

box size	N(gasket)	N(line segment)
1	1	1
$1/2$	3	2
$1/4 = 1/2^2$	$9 = 3^2$	4
$1/8 = 1/2^3$	3^3	$8 = 2^3$
$1/2^4$	3^4	2^4

$$N(1/2^n) = 3^n + 2^n$$

$$d = \lim_{n\to\infty} \frac{\log(N(1/2^n))}{\log(1/(1/2^n))} = \lim_{n\to\infty} \frac{\log(3^n+2^n)}{\log(2^n)}$$

$$= \lim_{n\to\infty} \frac{\log(3^n(1+2^n/3^n))}{\log(2^n)}$$

$$= \lim_{n\to\infty} \frac{\log(3^n) + \log(1+2^n/3^n)}{\log(2^n)}$$

$$= \lim_{n\to\infty} \left(\frac{\log(3^n)}{\log(2^n)} + \frac{\log(1+2^n/3^n)}{\log(2^n)} \right)$$

$$2^n/3^n = \left(\tfrac{2}{3}\right)^n \to 0$$

$$\frac{\log(1+2^n/3^n)}{\log(2^n)} \to 0$$

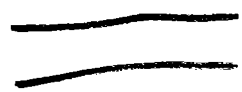
$$= \log 3/\log 2$$

If A and B lie in n-dimensional space, then for typical placements of A and B, $d(A \cap B) = d(A) + d(B) - n$

A & B are line segments in the plane

$d(A) = d(B) = 1$, $n = 2$

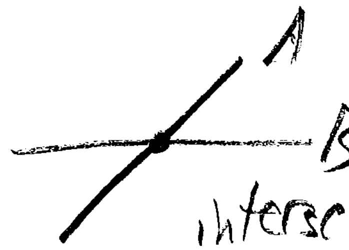
A is parallel to B (same slope)



no intersection

A and B coincide (same slope & pass through the same point)

intersection is a line segment



intersection is a point

$$d(A \cap B) = d(A) + d(B) - n$$
$$= 1 + 1 - 2 = 0$$

A & B are line segments in space $n = 3$

$$d(A \cap B) = d(A) + d(B) - 3$$
$$= 1 + 1 - 3 = -1$$

negative dimension means the intersection is empty

A & B planes , $n = 3$

$d(A \cap B) = 2 + 2 - 3 = 1$