

Review the calculation of the boxcounting dimension of the gasket. In general, $N(r)$ is the number of boxes of side length r needed to cover the shape. Then $N(r) \sim (1/r)^d$, so

$$d = \lim_{r \rightarrow 0} \log(N(r)) / \log(1/r)$$

The symmetry of the gasket suggests taking $r = 1/2, 1/4, 1/8, \dots, 1/2^n$. We see $N(1/2) = 3$,

$$N(1/4) = 9, \text{ that is, } N(1/2^2) = 3^2$$

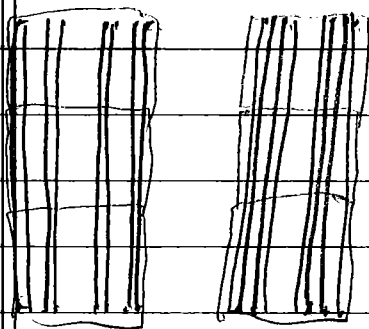
$$N(1/8) = 27, \text{ that is, } N(1/2^3) = 3^3$$

and in general, $N(1/2^n) = 3^n$. Here $r = 1/2^n$, so $r \rightarrow 0$ means $n \rightarrow \infty$

$$\text{We find } d = \lim_{n \rightarrow \infty} \log(N(1/2^n)) / \log(1/(1/2^n))$$

$$= \lim_{n \rightarrow \infty} \log(3^n) / \log(2^n) = \lim_{n \rightarrow \infty} n \log 3 / n \log 2 = \log 3 / \log 2$$

Now take the product of a Cantor set in the x -direction, and the unit interval in the y -direction.



The symmetry of the Cantor set suggests using squares of side length $1/3, 1/3^2, 1/3^3, \dots$

$$N(1/3) = 6$$

$$N(1/3^2) = 6^2$$

$$N(1/3^n) = 6^n$$

$$d = \lim_{n \rightarrow \infty} \frac{\log(N(1/3^n))}{\log(1/(1/3^n))} = \lim_{n \rightarrow \infty} \frac{\log(6^n)}{\log(3^n)} = \lim_{n \rightarrow \infty} \frac{n \log 6}{n \log 3} = \frac{\log 6}{\log 3}$$

$$= \frac{\log(2 \cdot 3)}{\log 3} = \frac{\log 2}{\log 3} + \frac{\log 3}{\log 3} = \frac{\log 2}{\log 3} + 1 = d(\text{Cantor set}) + d(\text{interval})$$