Cover a shape with boxes of side length r, say $N(r)$ is the minimum number of boxes needed.

power law hypothesis is $N(r) \simeq (1/r)^d$

or $\boxed{N(r) = k \cdot (1/r)^d}$

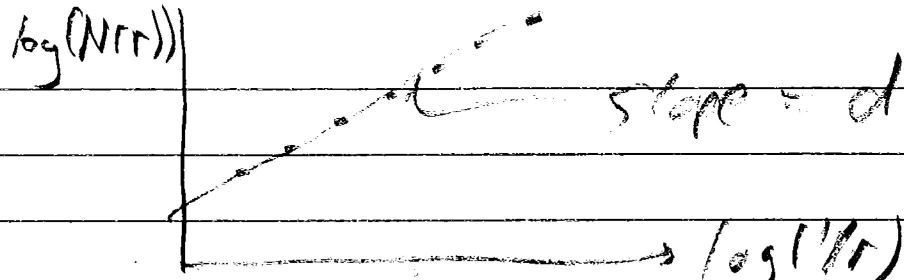
If this holds, then d is the <u>box-counting dimension</u> of the shape.

Facts about logarithms:

1. Logs turn products into sums
   $\mathrm{Log}(a \cdot b) = \log(a) + \log(b)$
2. Logs turn exponents into factors
   $\mathrm{Log}(a^b) = b \, \mathrm{Log}(a)$
3. To compute logs, use the log button on your calculator.

$$\log(N(r)) = \log(k \cdot (1/r)^d)$$
$$= \log(k) + \log((1/r)^d)$$
$$= \log(k) + d \log(1/r)$$

$$\log(N(r)) = d \cdot \log(1/r) + \log(k)$$
$$y = m \cdot x + b$$



If we can find a formula for the number of boxes, N(r), then

$$\log(N(r)) = \log(k) + d\log(1/r)$$

divide by $\log(1/r)$

$$\frac{\log(N(r))}{\log(1/r)} = \frac{\log(k)}{\log(1/r)} + d$$

As $r \to 0$, $1/r \to \infty$, and $\log(1/r) \to \infty$

so $\dfrac{\log(k)}{\log(1/r)} \to 0$

This gives $d = \lim\limits_{r \to 0} \dfrac{\log(N(r))}{\log(1/r)}$