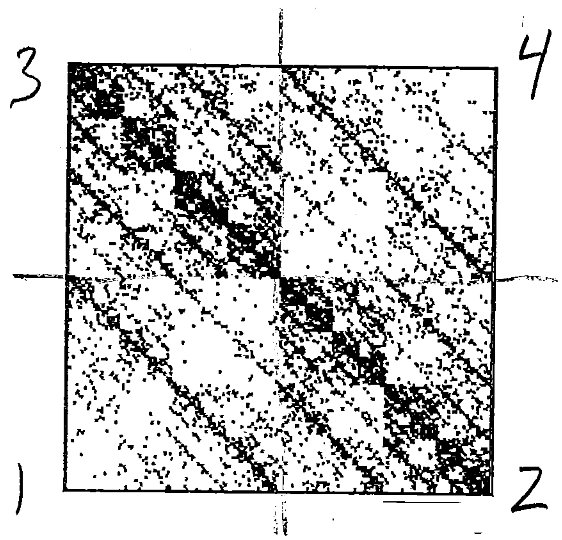


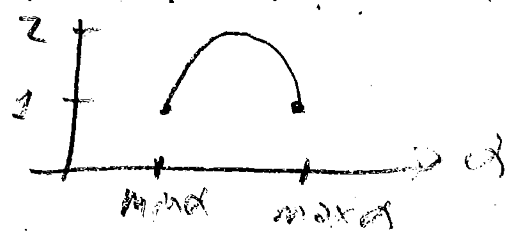
Because squares 2 and 3 appear to be about as filled, we say $p_2 = p_3$; similarly,

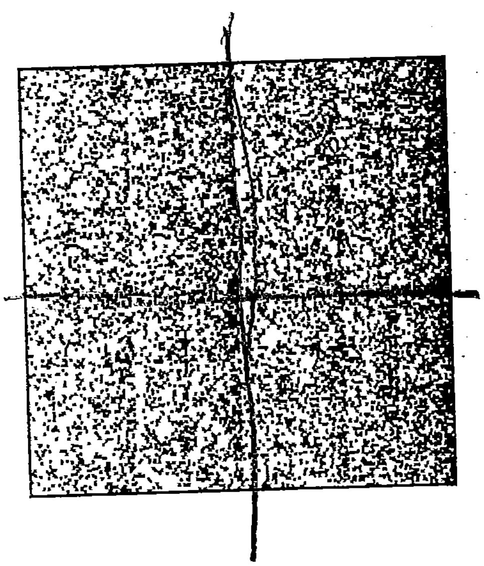
$p_1 = p_4 < p_2 = p_3$

$\min(\alpha) = \dfrac{\log(p_2)}{\log(1/2)}$ $\max(\alpha) = \dfrac{\log(p_1)}{\log(1/2)}$

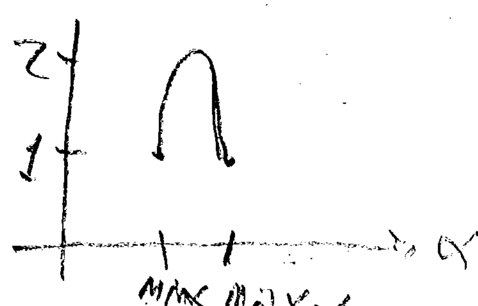
$\min(\alpha)$ occurs where T_2 and T_3 are applied; this is a line (between corner 2 and corner 3), so $f(\min(\alpha)) = \dim(\text{line}) = 1$

$\max(\alpha)$ occurs where T_1 and T_4 are applied, another line, so $f(\max(\alpha)) = \dim(\text{line}) = 1$. $\max(f(\alpha)) = 2$

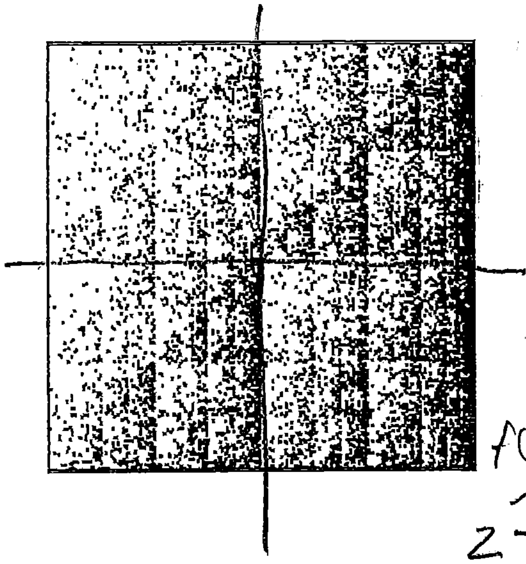




$p_2 = p_4 > p_1 = p_3$



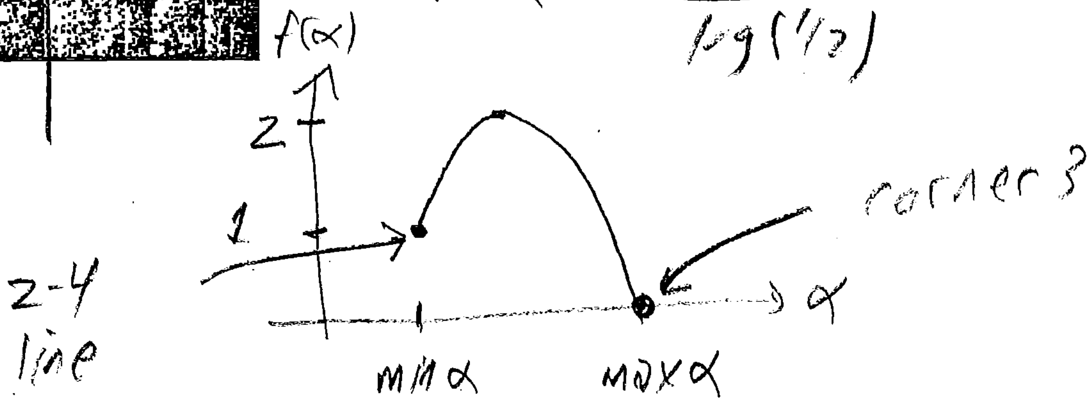
smaller range of probs gives a smaller range of αs.

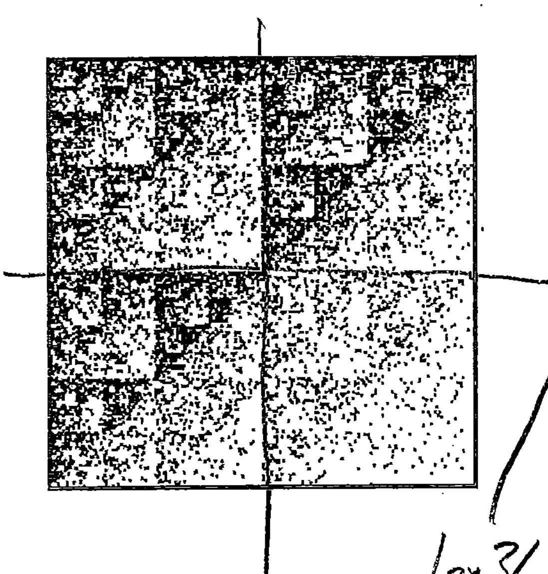


$p_2 = p_4 > p_1 > p_3$

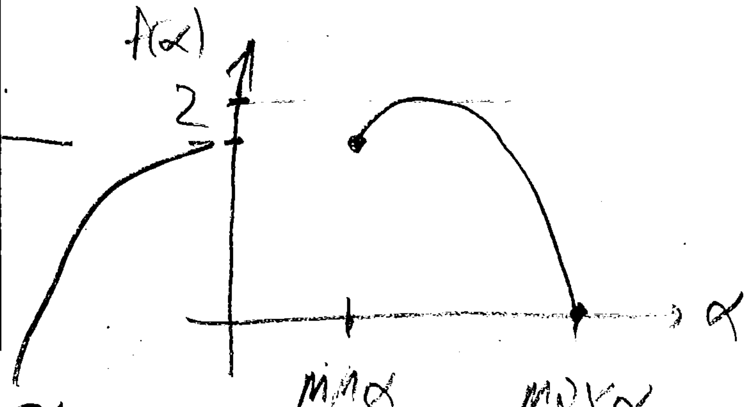
$\max\alpha = \frac{\log(p_3)}{\log(1/2)}$

$\min\alpha = \frac{\log(p_2)}{\log(1/2)}$

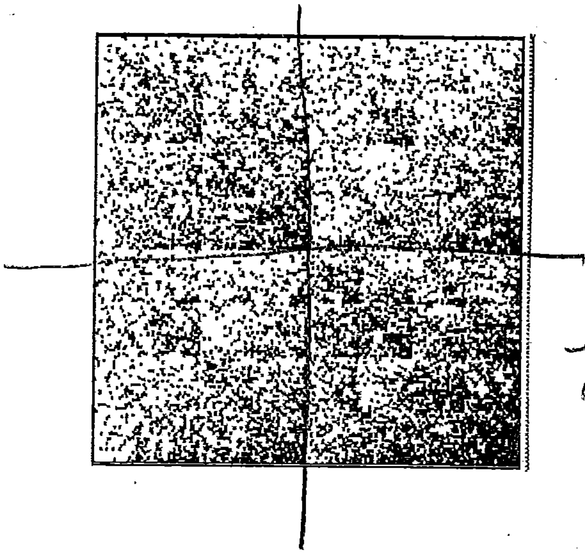




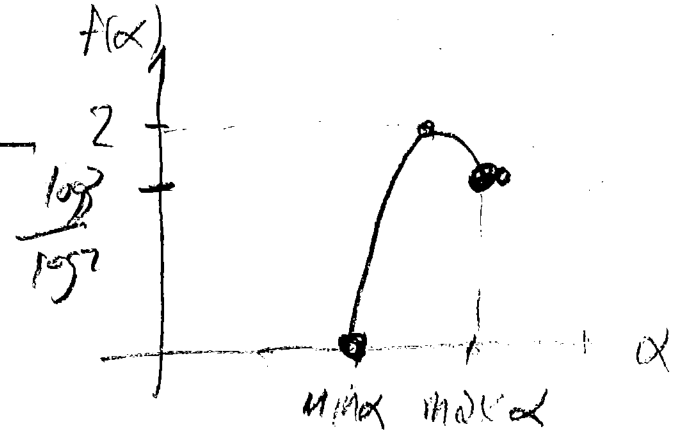
$p_1 = p_3 = p_4 > p_2$

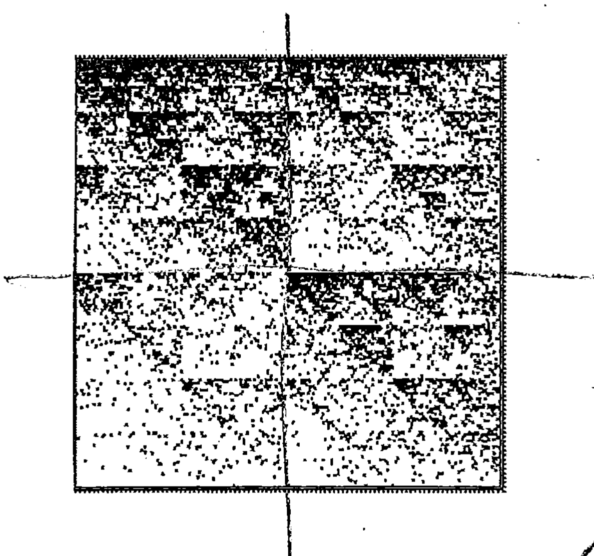


$\log 3/\log 2$
$= \dim(134 \text{ gasket})$

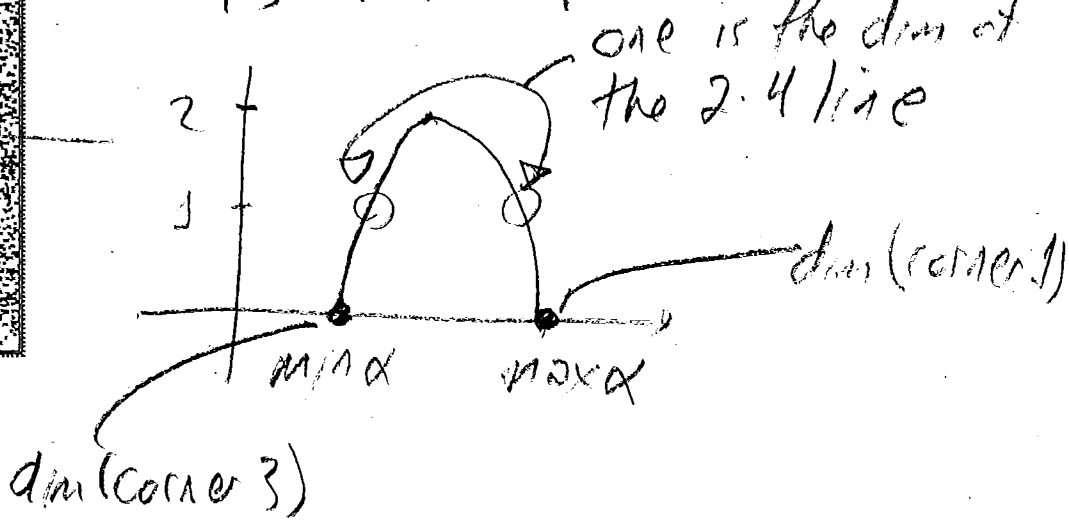


$p_2 > p_1 = p_3 = p_4$





$p_3 > p_2 = p_4 > p_1$



r	s	θ	φ	e	f	prob
1/2	1/2	0	0	0	0	.3
1/2	1/2	0	0	1/2	0	.3
1/2	1/2	0	0	0	1/2	.3
1/4	1/4	0	0	3/4	3/4	.1

$$\max(\alpha) = \frac{\log(.3)}{\log(1/2)} = 1.74 \qquad \frac{\log(.1)}{\log(1/4)} = 1.66 = \min(\alpha)$$

$$f(\max\alpha) = \dim\{T_1, T_2, T_3\} = \dim(\text{gasket}) = \log 3/\log 2$$

$$f(\min\alpha) = \dim\{T_4\} = \dim(\text{corner } 4) = 0$$

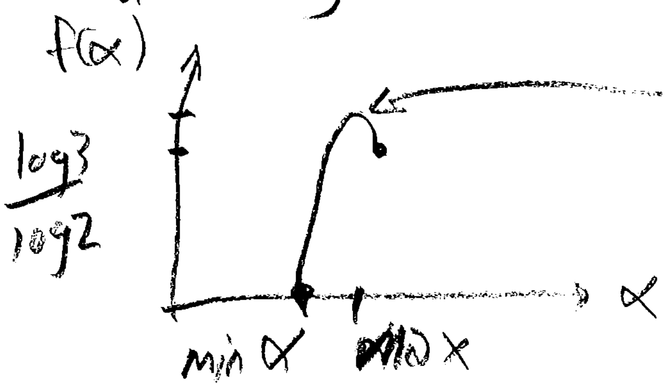
$$\max(f(\alpha)) = d : 3\left(\tfrac{1}{2}\right)^d + \left(\tfrac{1}{4}\right)^d = 1$$

$$\left(\tfrac{1}{2}\right)^d = x \qquad 3x + x^2 = 1$$

$$x^2 + 3x - 1 = 0 \quad x = \frac{-3 \pm \sqrt{3^2 - 4\cdot 1\cdot(-1)}}{2\cdot 1} = \frac{-3 \pm \sqrt{13}}{2}$$

$$\left(\tfrac{1}{2}\right)^d = \frac{-3 + \sqrt{13}}{2}$$

$$d = \log\left(\left(-3 + \sqrt{13}\right)/2\right) \Big/ \log(1/2) < 2$$



Randomized Cantor set:

with prob. $1/3$, replace each interval by 2 intervals scaled by $1/2$

with prob $2/3$, replace each interval by 2 intervals scaled by $\frac{1}{4}$

$1 = E(r_1^d + r_2^d)$ = "expected value" or "average"

$$= \frac{1}{3}\left(\frac{1}{2}^d + \frac{1}{2}^d\right) + \frac{2}{3}\left(\frac{1}{4}^d + \frac{1}{4}^d\right)$$

$$1 = \frac{2}{3}\cdot\left(\frac{1}{2}\right)^d + \frac{4}{3}\cdot\left(\frac{1}{4}\right)^d$$

$$x = \left(\frac{1}{2}\right)^d, \text{ so } \left(\frac{1}{4}\right)^d = x^2$$

$$1 = \frac{2}{3}x + \frac{4}{3}x^2$$

$$3 = 2x + 4x^2 \quad \cdots \quad x = \frac{-1+\sqrt{13}}{4}$$

$$d = \frac{\log\left(\frac{-1+\sqrt{13}}{4}\right)}{\log(1/2)}$$