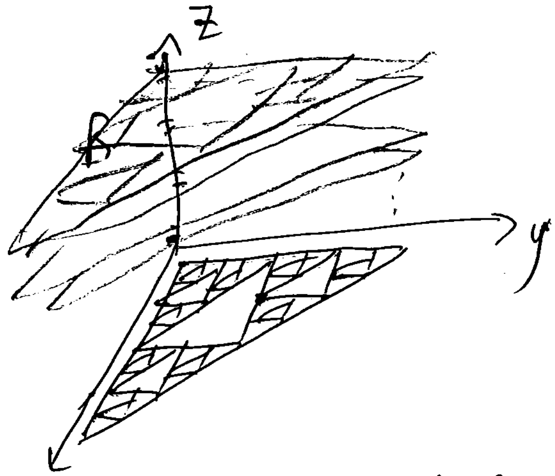
Review: algebra of dimensions

$\dim(A \times B) = \dim(A) + \dim(B)$

$\dim(A \cup B) = \max\{\dim(A), \dim(B)\}$

If A and B lie in n-dimensional space, then typically, $\dim(A \cap B) = \dim(A) + \dim(B) - n$



A = gasket in the xy plane

B = Cantor set along the z-axis

$$\dim(A \times B) = \dim(A) + \dim(B) = \frac{\log 3}{\log 2} + \frac{\log 2}{\log 3}$$

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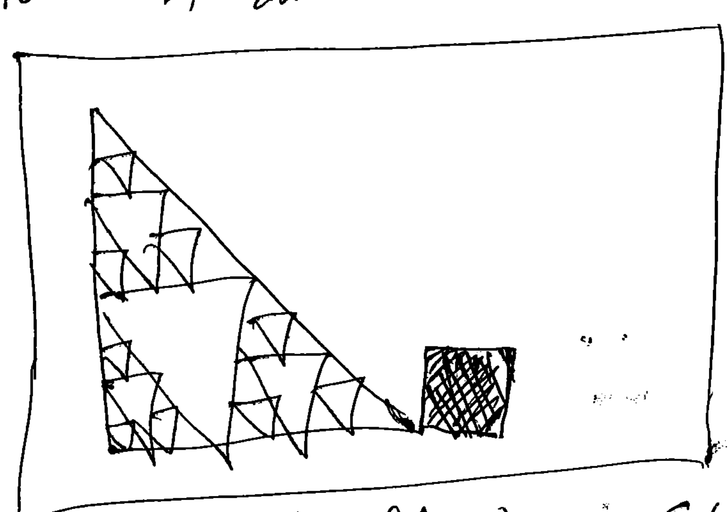
union A and B lie in the same space



A is the gasket

B is a filled in square

both lie in the xy-plane.

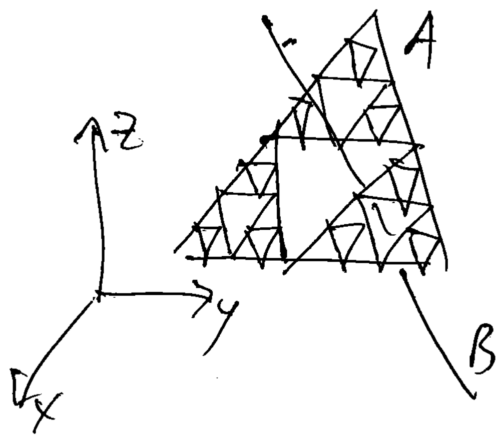
$\dim(A) = \log 3/\log 2$

$\dim(B) = 2$

$\dim(A \cup B) = \max\{\log 3/\log 2, 2\} = 2$

intersections A and B live in the same space, of dimension n.

$n = 3$



A = gasket

B = line segment

Typically $\dim(A \cap B) = \dim(A) + \dim(B) - 3$

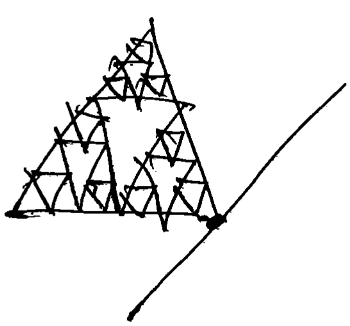
$$= \frac{\log 3}{\log 2} + 1 - 3$$

$$= \frac{\log 3}{\log 2} - 2 < 0$$

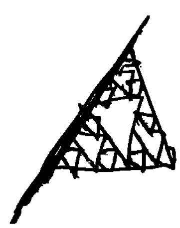
$$\boxed{\frac{\log 4}{\log 2} = 2, \text{ so } \frac{\log 3}{\log 2} < 2}$$

This means that typically the line segment passes entirely through a hole in the gasket.

Other intersections are possible:



here $A \cap B$ is a point, so $\dim(A \cap B) = \dim(\text{point}) = 0$



Here $A \cap B$ is a line segment, so $\dim(A \cap B) = \dim(\text{line}) = 1$