Hölder exponent

.3	.3
.1	.3

$\alpha(i)$ = measures the probability of that address, scaled with the box size

$$\text{Prob}(\text{address } i) = \left(\tfrac{1}{2}\right)^{\alpha(i)}$$

$$\log(\text{Prob}(i)) = \log\left(\left(\tfrac{1}{2}\right)^{\alpha(i)}\right)$$

$$= \alpha(i)\cdot\log\left(\tfrac{1}{2}\right)$$

$$\alpha(i) = \frac{\log(\text{Prob}(i))}{\log(1/2)}$$

$$\alpha(1) = \frac{\log(\text{Prob}(1))}{\log(1/2)} = \frac{\log(.1)}{\log(.5)} = 3.32$$

$$\alpha(2) = \alpha(3) = \alpha(4) = \frac{\log(\text{Prob}(2))}{\log(1/2)} = \frac{\log(.3)}{\log(.5)} = 1.74$$

If all pieces have the same scaling,
higher prob = lower α
lower prob = higher α.

.09	.09	.09	.09
.03	.09	.03	.09
.03	.03	.09	.09
.01	.03	.03	.09

$$\alpha(11) = \frac{\log(\text{Prob}(11))}{\log(1/4)} = \frac{\log(.01)}{\log(1/4)}$$

$$= \frac{\log(.1^2)}{\log((1/2)^2)} = \frac{2\log(.1)}{2\log(.5)} = 3.32$$

Maximum value of α is $\dfrac{\log(\text{min prob})}{\log(1/2)}$

minimum value of α is $\dfrac{\log(\text{max prob})}{\log(1/2)}$

$$\alpha_{min} = \frac{\log(.09)}{\log(.25)} = \frac{\log(.3^2)}{\log(.5^2)} = \frac{2\log(.3)}{2\log(.5)}$$

$$= 1.74$$

$$\alpha(\text{square}) = \frac{\log(\text{prob of landing in a square})}{\log(\text{side length of the square})}$$

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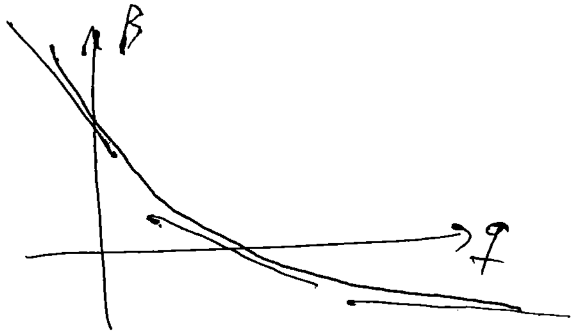
generalized Moran equation:

$$r_1^d + r_2^d + \cdots + r_N^d = 1 \quad \left(\text{regular Moran equation}\right)$$

$$p_1^q r_1^{\beta(q)} + p_2^q r_2^{\beta(q)} + \cdots + p_N^q r_N^{\beta(q)} = 1$$

$\underline{-\infty < q < \infty}$ large $+q$ emphasizes max prob.

large $-q$ emphasizes min prob.



$\alpha(q)$ = negative of the slope of the tangent to $\beta(q)$

[lot of work]

$$f(\alpha(q)) = q \cdot \alpha(q) + \beta(q)$$

What happens when $q = 0$?

gen. Moran: $p_1^0 r_1^{\beta(0)} + p_2^0 r_2^{\beta(0)} + \dots + p_N^0 r_N^{\beta(0)} = 1$

$$r_1^{\beta(0)} + r_2^{\beta(0)} + \dots + r_N^{\beta(0)} = 1$$

This is the regular Moran equation, so $\beta(0)$ = dimension of the shape.

$f(\alpha(0)) = 0 \cdot \alpha(0) + \beta(0)$

$f(\alpha(0)) = \beta(0)$ = dimension of the shape.

The maximum point on the $f(\alpha)$ curve occurs where $df/d\alpha = 0$; $df/d\alpha = q$

so max of the $f(\alpha)$ curve occurs when $q = 0$