

Chaos - complicated behavior from simple sources.

sensitivity to initial conditions - "Even the tiniest change can alter the future in ways you can't imagine"

A. Simpson

↳ butterfly effect: the flapping of a butterfly's wings in Texas can cause a tornado in Kansas.

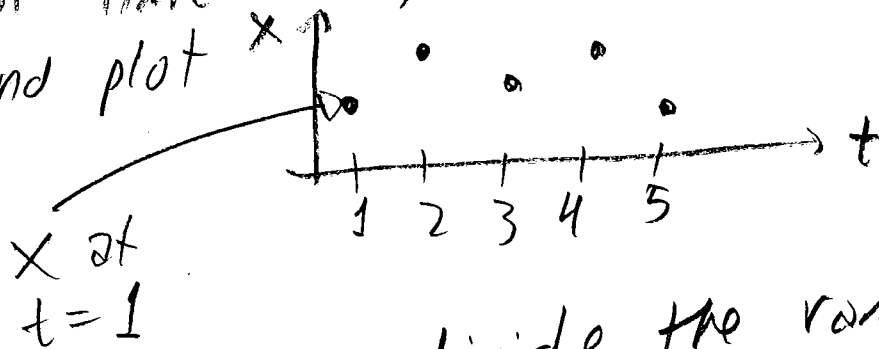
This is incorrect: a loss of our ability to predict the future is the effect of chaos.

## Methods of presenting data

time series

measure at times  $t=1, t=2, t=3, t=4, \dots$

and plot  $x$



histograms

divide the range of  $x$ -values into bins and count the number of times each bin is visited

return maps

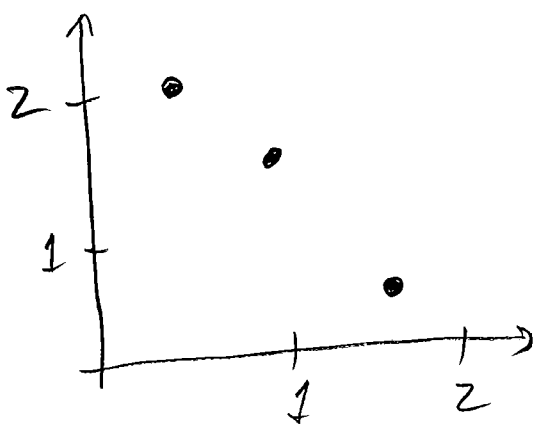
Given the sequential measurements

$x_1, x_2, x_3, x_4, \dots$

↳  $x$  at time  $t-1$

plot the points  $(x_1, x_2), (x_2, x_3), (x_3, x_4), \dots$

If  $x_1 = 1, x_2 = 1.5, x_3 = .5, x_4 = 2, \dots$



$(x_1, x_2) = (1, 1.5)$

$(x_2, x_3) = (1.5, .5)$

$(x_3, x_4) = (.5, 2)$

If each  $x_i$  is the result of applying a function (logistic map, tent map, or something else) to  $x_{i-1}$ , then the points of the return map will lie on the graph of the function.

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Today: driven IFS plots & detecting forbidden combinations  
 Kelly maps  
 ↳ detecting repeating patterns