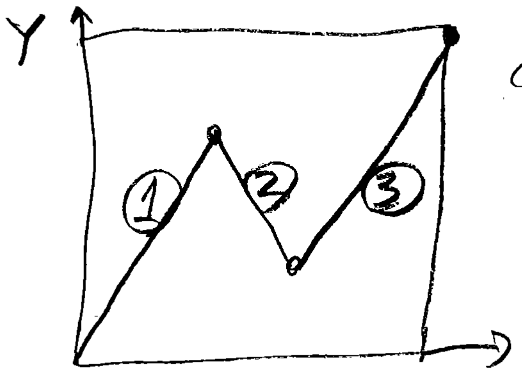
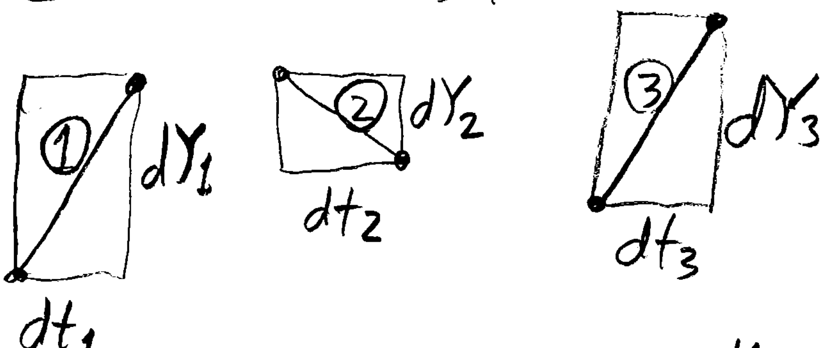
Review: finance cartoons, basic constructions of chaos



cartoon generator



Power-law scaling in the model

$$|dY_1| = (dt_1)^{H_1}$$
$$|dY_2| = (dt_2)^{H_2}$$
$$|dY_3| = (dt_3)^{H_3}$$

Find H_1

$$\log |dY_1| = \log\left((dt_1)^{H_1}\right) = H_1 \log(dt_1)$$

$$H_1 = \frac{\log |dY_1|}{\log(dt_1)}$$

Similarly, $H_2 = \dfrac{\log |dY_2|}{\log(dt_2)}$

$$H_3 = \frac{\log |dY_3|}{\log(dt_3)}$$

If $H_1 = H_2 = H_3$, this is a unifractal.
If not, it is a multifractal

Trading time theorem

convert a multifractal cartoon into a unifractal cartoon in trading time, time rescaled as a multifractal.

Take the vertical jumps

$$dY_1, dY_2, dY_3, \ldots, dY_N$$

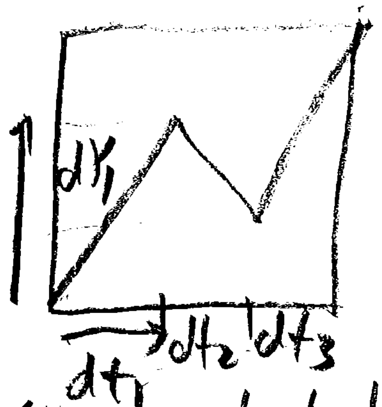
of the generator. Find the solution D of

$$|dY_1|^D + |dY_2|^D + |dY_3|^D + \cdots + |dY_N|^D = 1$$

solve for D. Then the trading time generators are

$$\begin{aligned} dT_1 &= |dY_1|^D \\ dT_2 &= |dY_2|^D \\ dT_3 &= |dY_3|^D \\ &\vdots \\ dT_N &= |dY_N|^D \end{aligned}$$

The trading time expands the time scale during high movement periods and contracts time during low movement periods.

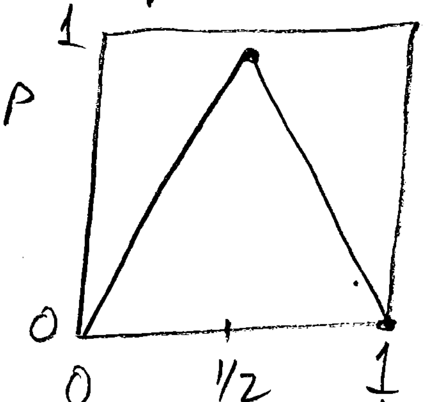


Y vs. clock time

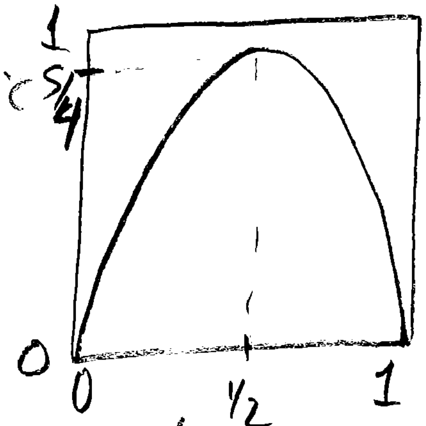
Fractals build complex shapes by iterating simple rules

chaos generates complex behavior by iterating simple rules.

Tent map



Logistic map

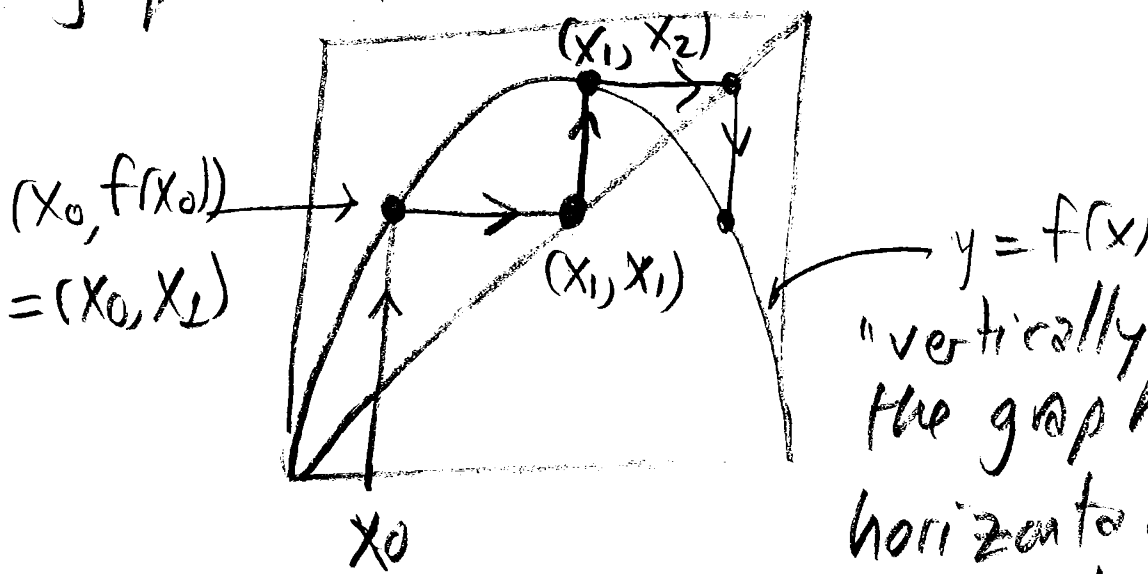


Say $f(x)$ denotes the tent map or the logistic map - e.g. $f(x) = s\,x(1-x)$ (logistic)

Given an initial value x_0, $x_1 = f(x_0)$, $x_2 = f(x_1)$, $x_3 = f(x_2)$, ... This is the <u>orbit</u> of x_0

Can we predict properties of the orbit without doing too much arithmetic?

graphical iteration



"vertically to the graph, horizontally to the diagonal."