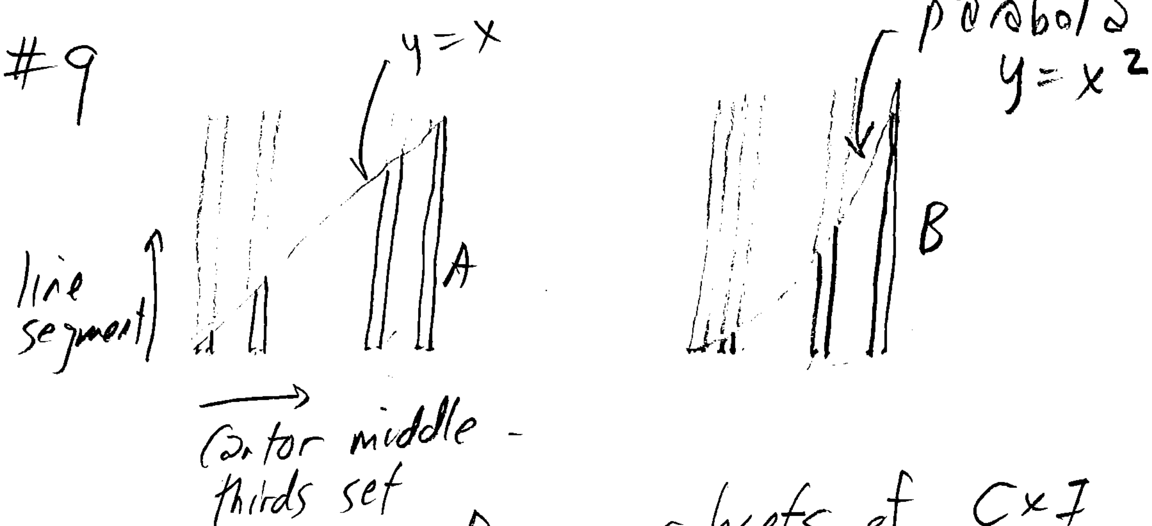
PF6 #9



Both A and B are subsets of $C \times I$

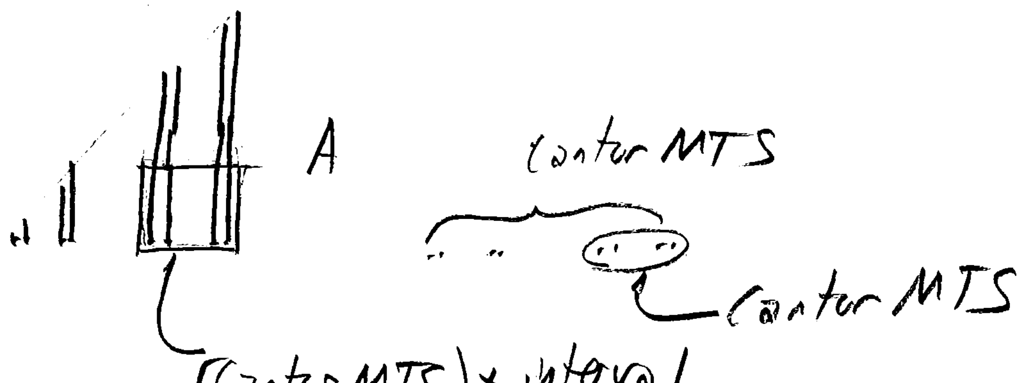
Monotonicity of dimension

If X is a subset of Y, then

$\dim(X) \le \dim(Y)$

$\dim(A) \le \dim(C \times I) = \dim(C) + \dim(I) = \frac{\log 2}{\log 3} + 1$

$\dim(B) \le \dim(C \times I) = \dim(C) + \dim I = \frac{\log 2}{\log 3} + 1$



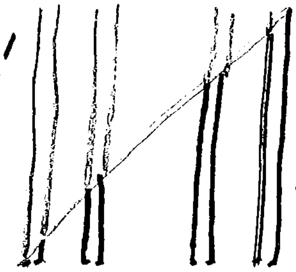
Call the piece in the box D. D is the product of a Cantor MTS and an interval,

so $\dim(D) = \frac{\log 2}{\log 3} + 1$

Because D is a subset of A, $\dim(D) \le \dim(A)$

$\frac{\log 2}{\log 3} + 1 = \dim(D) \le \dim(A) \le d(C \times I) = \frac{\log 2}{\log 3} + 1$

Then $\dim(A) = \frac{\log 2}{\log 3} + 1$



$$A \cup A' = C \times I$$
$$\dim(A \cup A') = \dim(C \times I)$$
$$= \dim(C) + \dim(I)$$
$$= \frac{\log 2}{\log 3} + 1$$

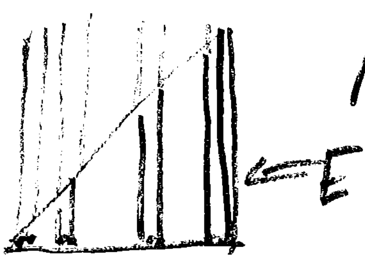
We know $\dim(A \cup A') = \max\{\dim(A), \dim(A')\}$

A' is just A rotated $180°$, so

$\dim(A) = \dim(A')$

Then $\frac{\log 2}{\log 3} + 1 = \max\{\dim(A), \dim(A')\}$

$$= \max\{\dim(A), \dim(A)\}$$
$$= \dim(A)$$

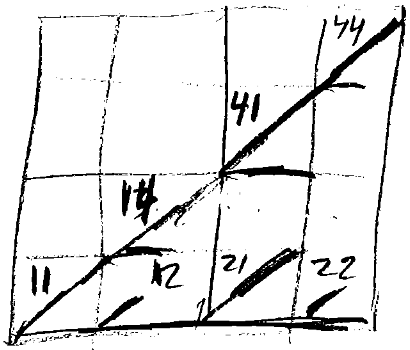


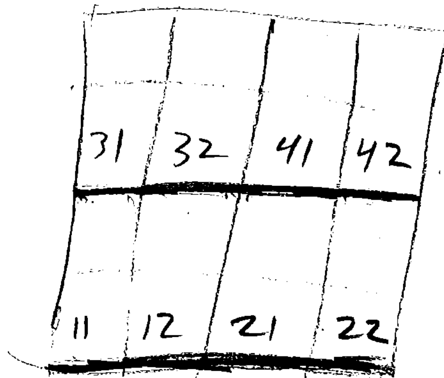
$A = (C \times I) \cap$ right triangle E

$$\dim(A) = \dim(C \times I) + \dim(E) - 2$$
$$= \dim(C \times I) + 2 - 2$$
$$= \dim(C \times I)$$

A and E lie in the plane

PF 5 #9

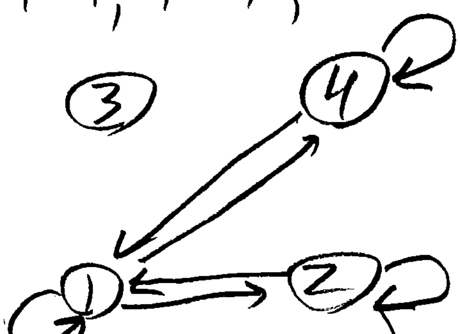


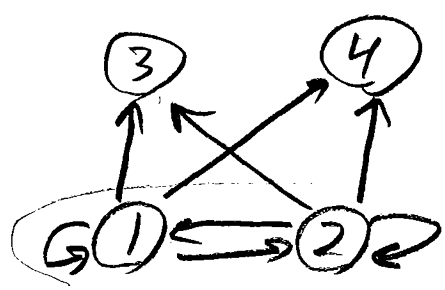


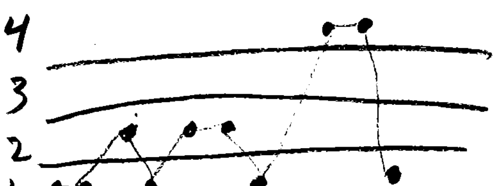
From the occupied addresses, find the allowed transitions

$1\to1,\ 2\to1,\ 1\to2,\ 2\to2$
$4\to1,\ 1\to4,\ 4\to4$

$1\to1,\ 2\to1,\ 1\to2,\ 2\to2$
$1\to3,\ 2\to3,\ 1\to4,\ 2\to4$

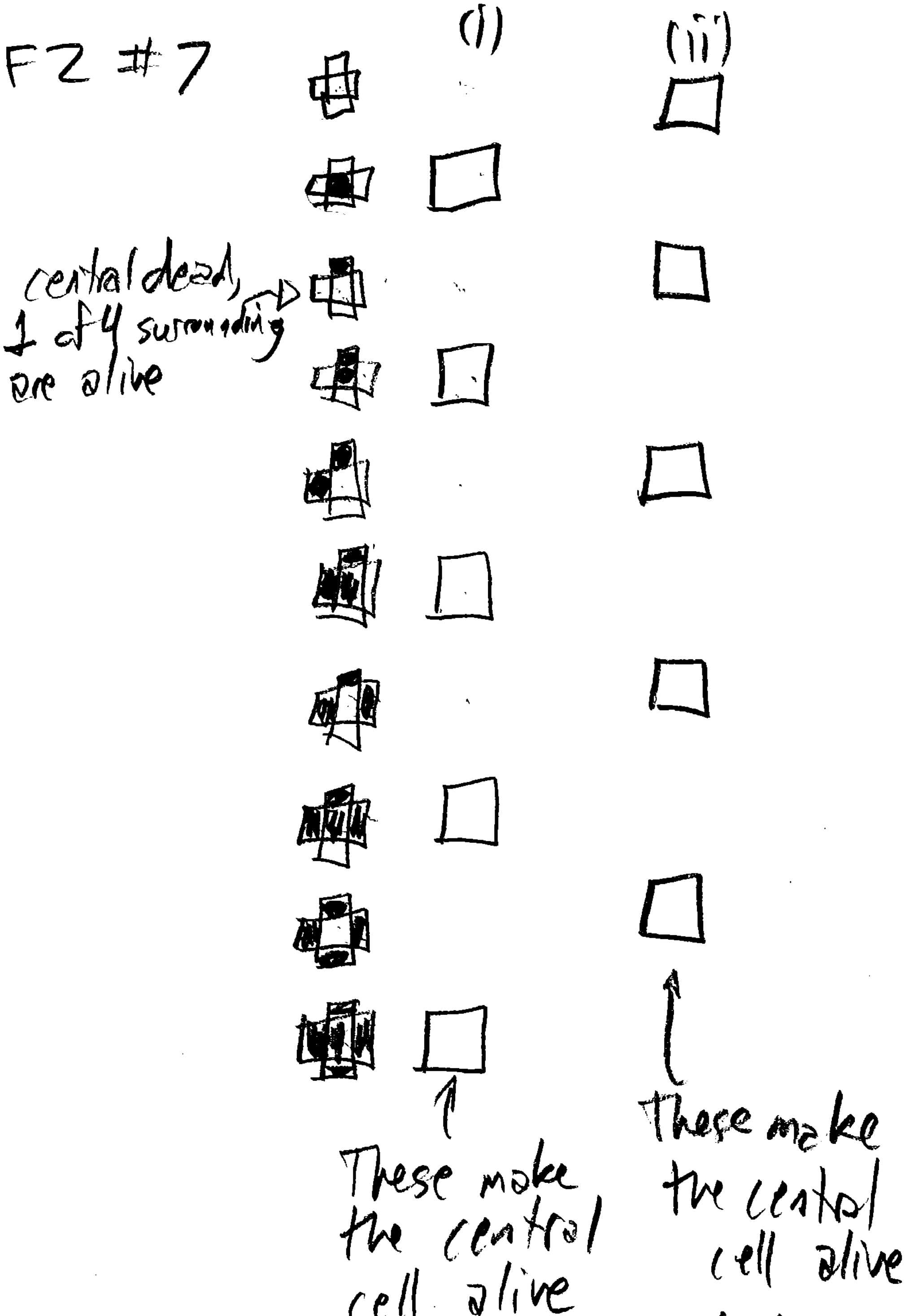






Can't continue from here because in the transition graph, nothing follows 3.

PFZ #7

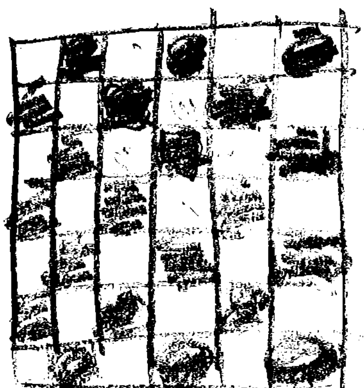


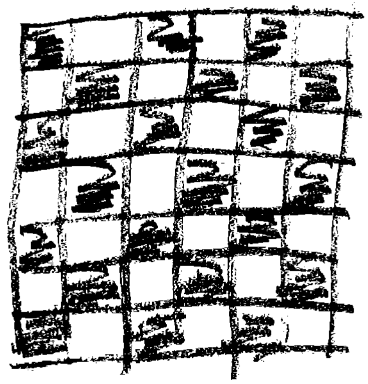
Live & dead cells reverse roles with each generation

rule (i) says if the central cell is alive, it stays alive; if the central cell is dead, it stays dead. Nothing changes.

These nbhds give a live central cell 

All others give dead central cells





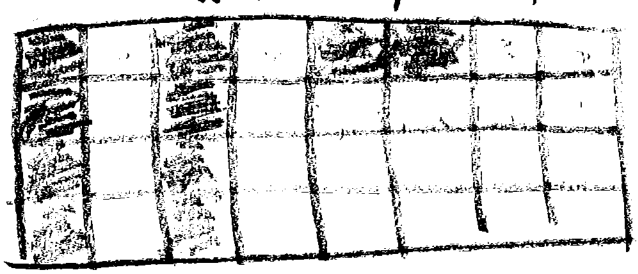
We have not selected , a live central cell with 4 dead neighbors dies in the next generation.

Because we have selected , a dead central cell with 4 live neighbors becomes alive in the next generation

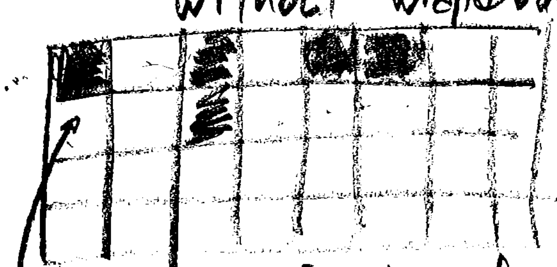
---

1 dim N=3 CA rule  gives live
all others give dead

with wraparound



without wraparound



(not infinite)

dies because it has no left neighbor

PF 4 #3 $\varepsilon = 1/2^n$

$N(\varepsilon) = 2^n + 3^n + n$

Find the box-counting dimension, given this information

$$d = \lim_{n\to\infty} \frac{\log(N)}{\log(1/\text{box size})} = \lim_{n\to\infty} \frac{\log(2^n+3^n+n)}{\log(1/(1/2^n))}$$

$$= \lim_{n\to\infty} \frac{\log\left(3^n \cdot \left(\frac{2^n}{3^n} + 1 + n/3^n\right)\right)}{\log(2^n)}$$ factor out the largest term.

$$= \lim_{n\to\infty} \frac{\log(3^n) + \log\left(\left(\frac{2}{3}\right)^n + 1 + \frac{n}{3^n}\right)}{\log(2^n)}$$

What happens to $\left(\frac{2}{3}\right)^n + 1 + \frac{n}{3^n}$ as $n \to \infty$?

As $n\to\infty$ $\left(\frac{2}{3}\right)^n \to 0$

$\frac{1}{3}, \frac{2}{9}, \frac{3}{27}, \frac{4}{81}, \frac{5}{243}, \ldots$

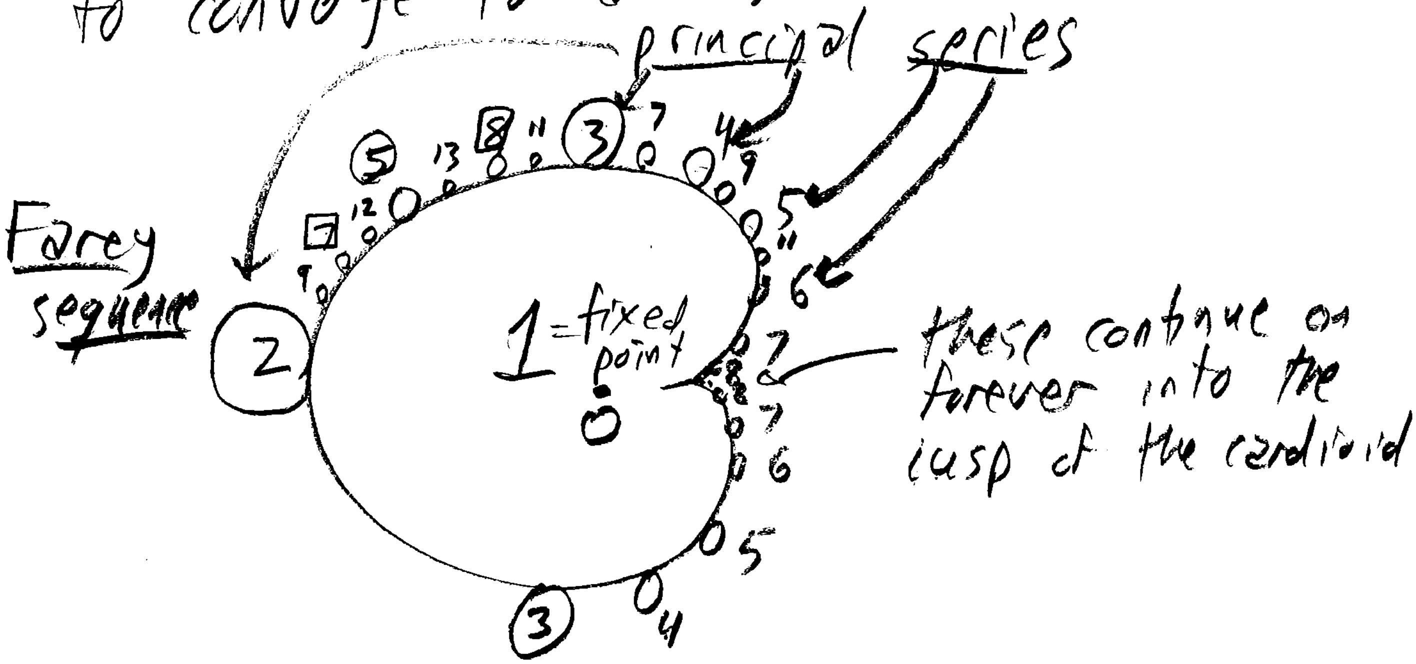
As $n\to\infty$ $\frac{n}{3^n} \to 0$

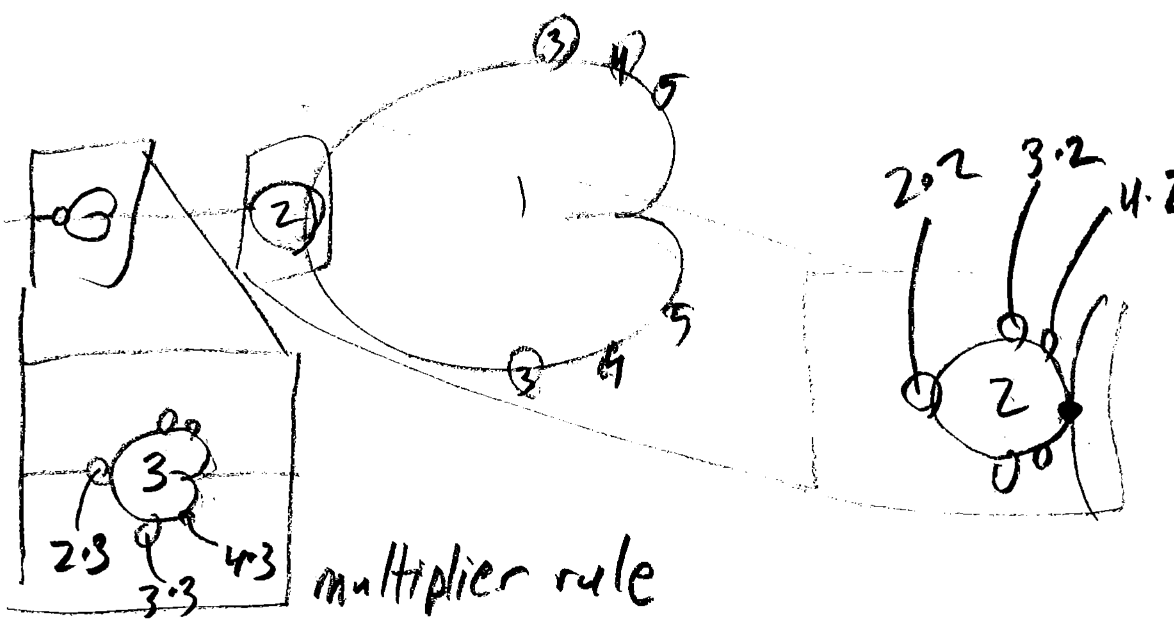
$$d = \lim_{n\to\infty} \frac{\log(3^n)}{\log(2^n)} + \frac{\log 1}{\log(2^n)}$$

$$= \lim_{n\to\infty} \frac{n\log 3}{n\log 2} + \frac{\log 1}{n\log 2} = \frac{\log 3}{\log 2}$$

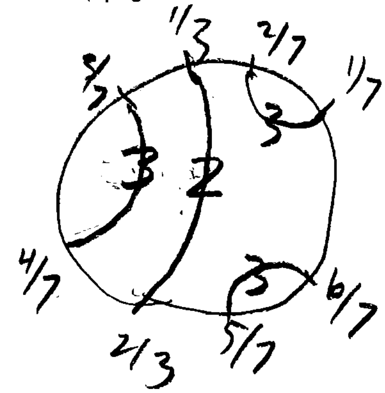
Mandelbasics

The Mandelbrot set is the collection of all c for which the iterates $z_1 = z_0^2 + c$, $z_2 = z_1^2 + c, \ldots$, starting from $z_0 = 0$, do not escape to ∞. One way this can happen is for the iterates to converge to a cycle.





To find the number of midget Mandelbrot sets, use Lavaurs' method.



cycle number

2 $\quad \frac{1}{2^2-1} = \frac{1}{3}, \frac{2}{3}$

3 $\quad \frac{1}{2^3-1} = \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ $\quad$ 3 pairs
3, 3-cycle pieces
2 discs, 1 midget

4

5

⋮

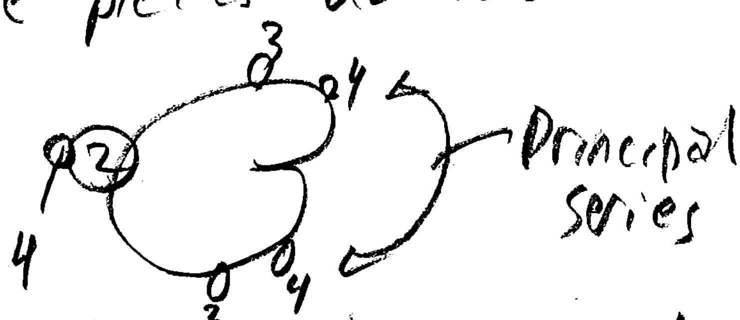
$$\frac{1}{2^4-1} = \frac{1}{15}\ \frac{2}{15}\ \frac{3}{15}\ \frac{4}{15}\ \boxed{\frac{5}{15}}\ \frac{6}{15}\ \frac{7}{15}\ \frac{8}{15}$$

$$\frac{9}{15}\ \boxed{\frac{10}{15}}\ \frac{11}{15}\ \frac{12}{15}\ \frac{13}{15}\ \frac{14}{15}$$

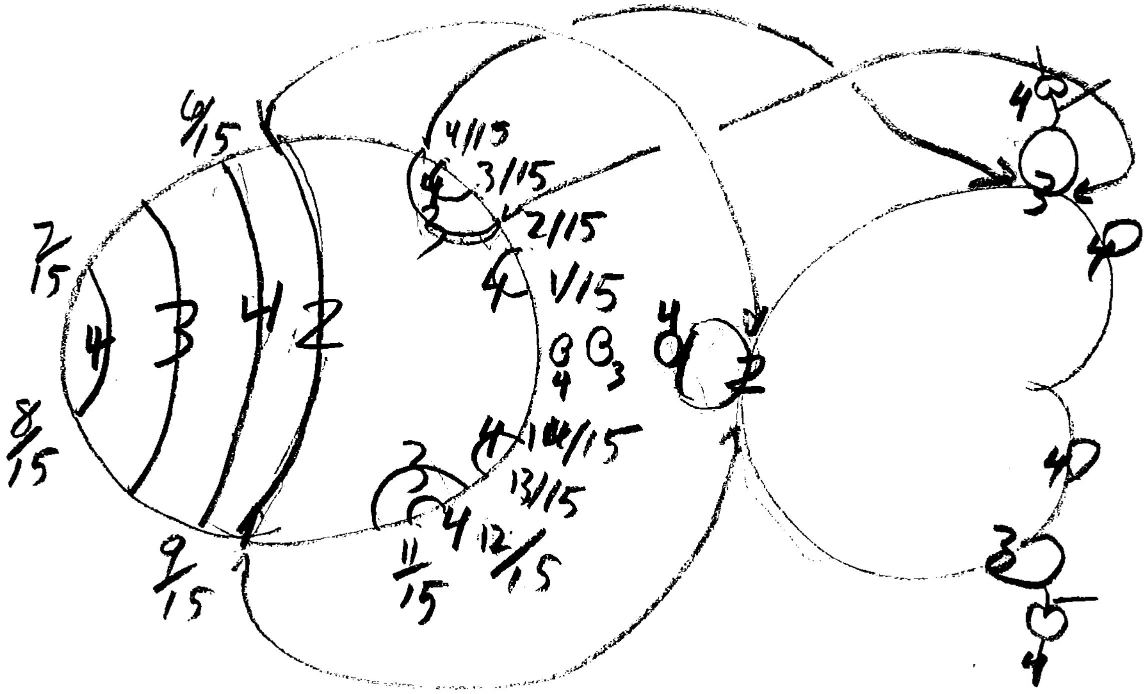
$\frac{5}{15} = \frac{1}{3}$ $\quad$ $\frac{10}{15} = \frac{2}{3}$

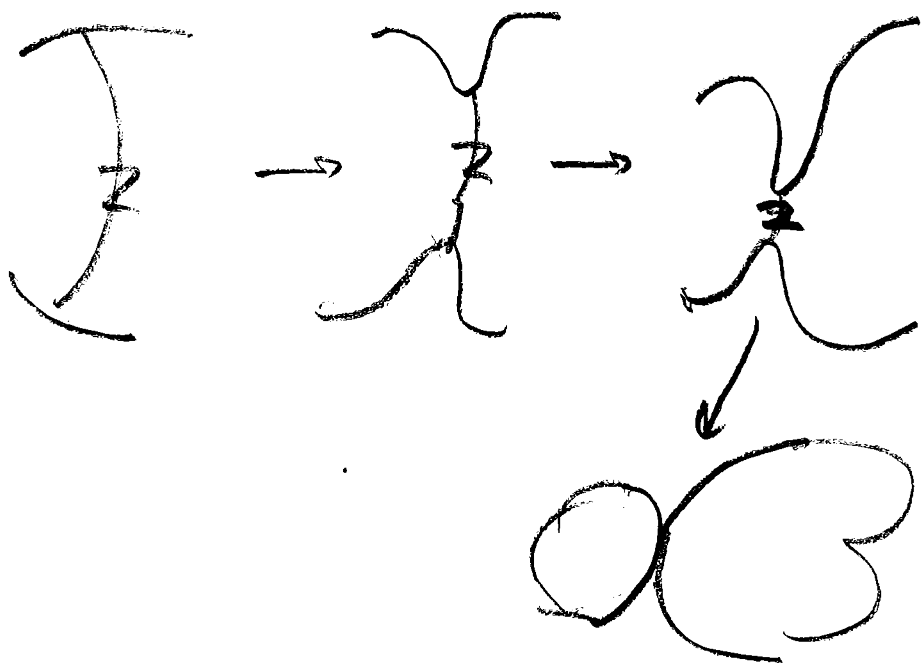
6 4-cycle pieces

What 4-cycle pieces do we know?

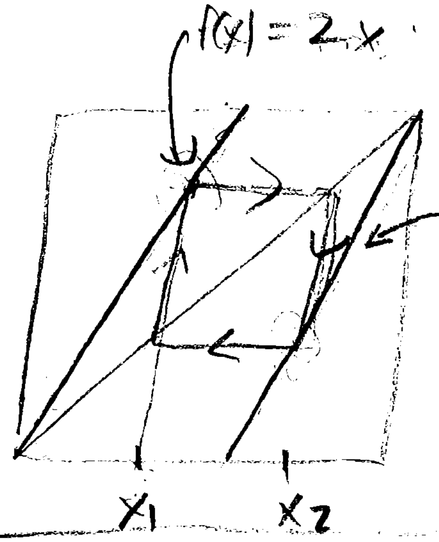


Three of the 6 4-cycle pieces are discs, so three must be midgets.


6/15
7/15
8/15
9/15
4
3
4
2
4/15
3/15
2/15
1/15
14/15
13/15
12/15
11/15



PF6 #8



Because x_1 and x_2 form a 2-cycle

$\boxed{f(x_1) = x_2 \text{ and } f(x_2) = x_1}$

$x_2 = f(x_1) = 2x_1$

$x_1 = f(x_2) = 2x_2 - 1$

Combine these:

$x_1 = 2x_2 - 1 = 2(2x_1) - 1$

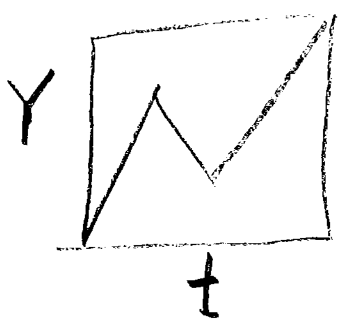
$x_1 = 4x_1 - 1$

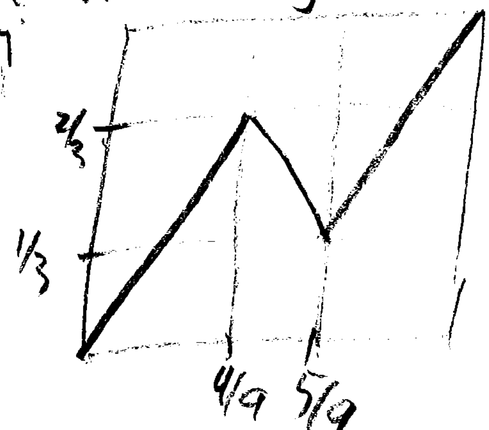
$-3x_1 = -1$

$x_1 = \frac{-1}{-3} = 1/3$ and $x_2 = 2x_1 = 2 \cdot \frac{1}{3} = \frac{2}{3}$

Brownian motion

① To recognize Brownian motion, for each piece of the generator $|\Delta Y| = \sqrt{\Delta t}$





$\Delta Y_1 = \frac{2}{3}, \Delta t_1 = 4/9$

$\Delta Y_2 = -\frac{1}{3}, \Delta t_2 = 1/9$

$\Delta Y_3 = \frac{2}{3}, \Delta t_3 = 4/9$

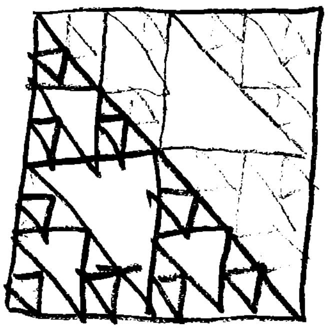
(2) In addition to $|\Delta Y| = \sqrt{\Delta t}$, Brownian motion has two other characteristics:

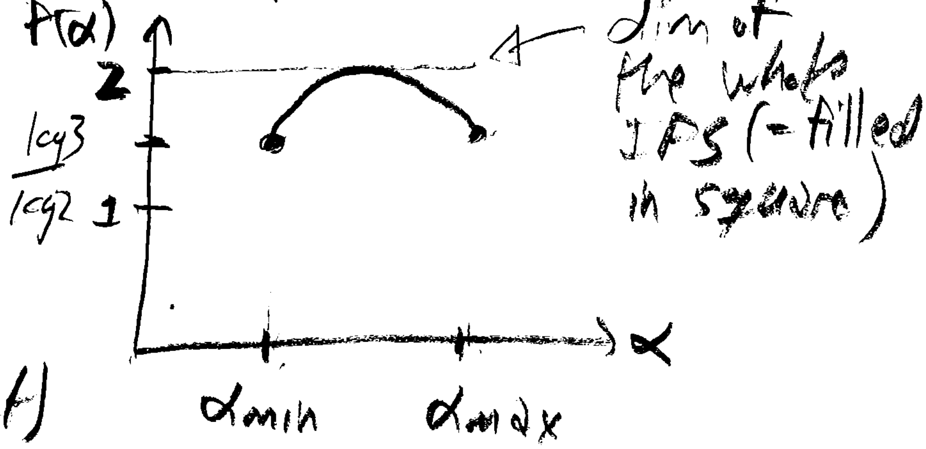
(i) jumps (increments) are independent of one another

(ii) jumps are normally distributed (follow the Bell curve), large jumps are exceedingly rare.

PF4 #5

dark gasket is the points with $\underline{\min \alpha}$, light gasket is the points with $\max \alpha$





$f(\alpha_{min}) =$ dim (dark gasket) $= \dfrac{\log 3}{\log 2}$

$f(\alpha_{max}) =$ dim (light gasket) $= \dfrac{\log 3}{\log 2}$

~~Suppose~~ Can this ~~be~~ be generated by

	r	s	θ	φ	e	f
T_1	.5	.5	0	0	0	0
T_2	.5	.5	0	0	.5	0
T_3	.5	.5	0	0	0	.5
T_4	.5	.5	0	0	.5	.5

All scaling factors are the same, so

min $\alpha \longleftrightarrow$ max prob

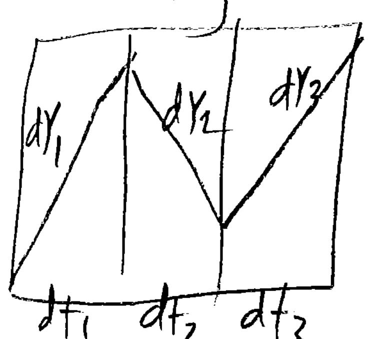
max $\alpha \longleftrightarrow$ min prob

min α occurs on the T_1, T_2, T_3 gasket,
so T_1, T_2, and T_3 must have max. prob.

max α occurs on the T_2, T_3, T_4 gasket,
so T_2, T_3, and T_4 must have min. prob.

But T_2 and T_3 cannot have both the highest and the lowest probabilities,
so this $f(\alpha)$ curve cannot be generated by these 4 transformations.

Trading Time Theorem



unifractal or multifractal?

$$\frac{\log|dY_1|}{\log dt_1} = H_1 \qquad \frac{\log|dY_2|}{\log dt_2} = H_2$$

$$\frac{\log|dY_3|}{\log dt_3} = H_3$$

unifractal if $H_1 = H_2 = H_3$

at least two differ make this a multifractal.

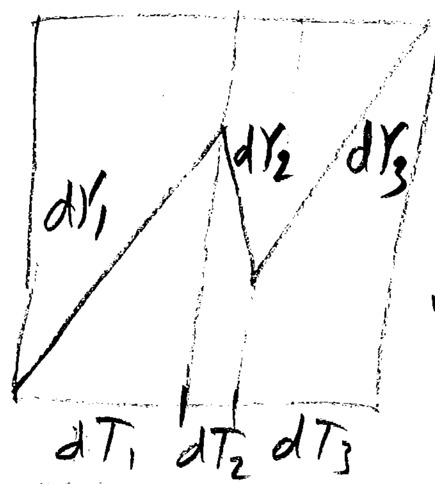
To apply the trading time theorem, solve

$$|dY_1|^D + |dY_2|^D + |dY_3|^D = 1$$

Sometimes we can solve for D by the Moran equation.

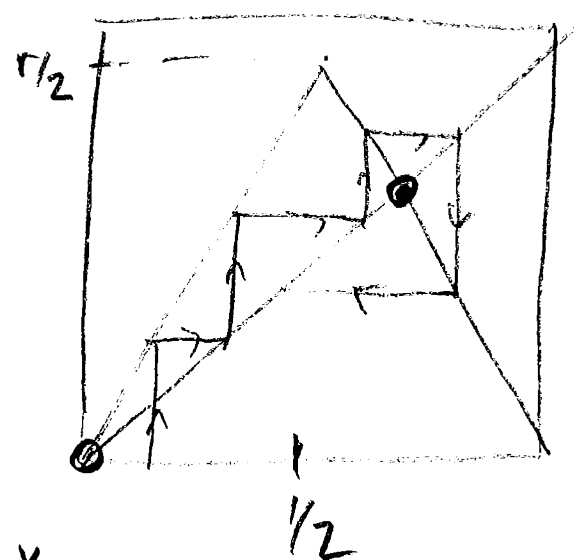
The Trading time generators are

$$dT_1 = |dY_1|^D$$
$$dT_2 = |dY_2|^D$$
$$dT_3 = |dY_3|^D$$



unifractal

Tent Map



graphical iteration on the tent.

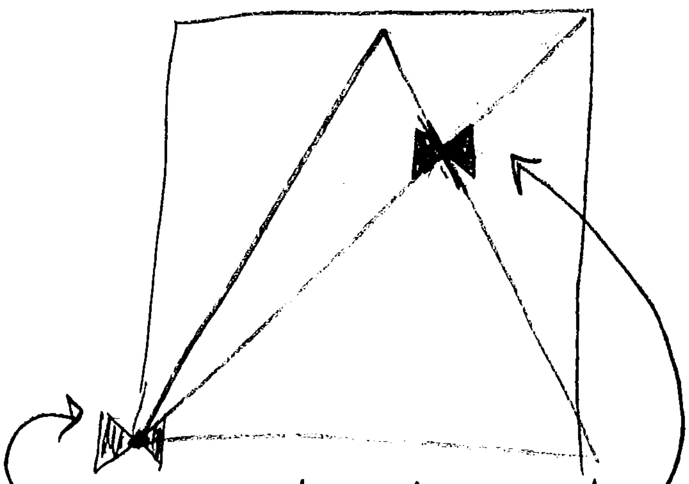
$x \le 1/2$

$T(x) = r \cdot x$

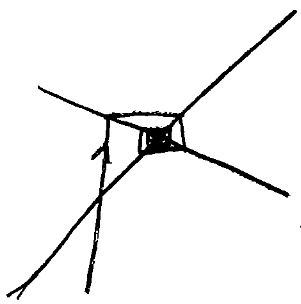
$x \geq 1/2$

$T(x) = r - r \cdot x$

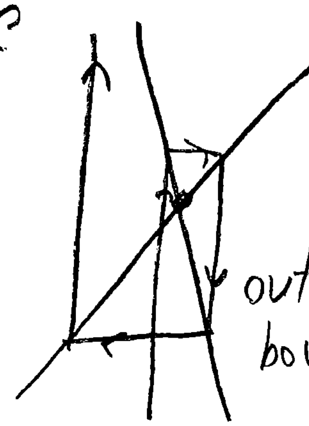
fixed points are the intersections of the graph and the diagonal



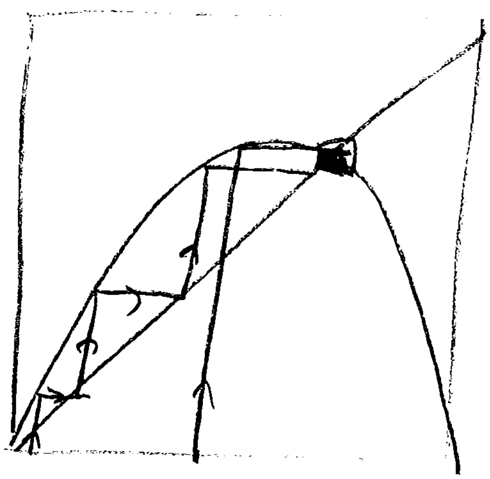
graph crosses the diagonal
outside the bow tie, so
the fixed point is unstable



inside
the bow
tie



outside the
bow tie



Both tent and
logistic have their
maximum values
above $x = 1/2$

$T(\frac{1}{2}) = r \cdot \frac{1}{2} = \frac{r}{2}$

$L(\frac{1}{2}) = r \cdot \frac{1}{2}(1 - \frac{1}{2})$

$= r \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{r}{4}$

$$T(x) = \begin{cases} rx & \text{for } x \le 1/2 \\ r - rx & \text{for } x \geq 1/2 \end{cases}$$

$L(x) = rx(1-x)$

For each c the Julia set J_c is the collection of those z_0 values for which the iterates

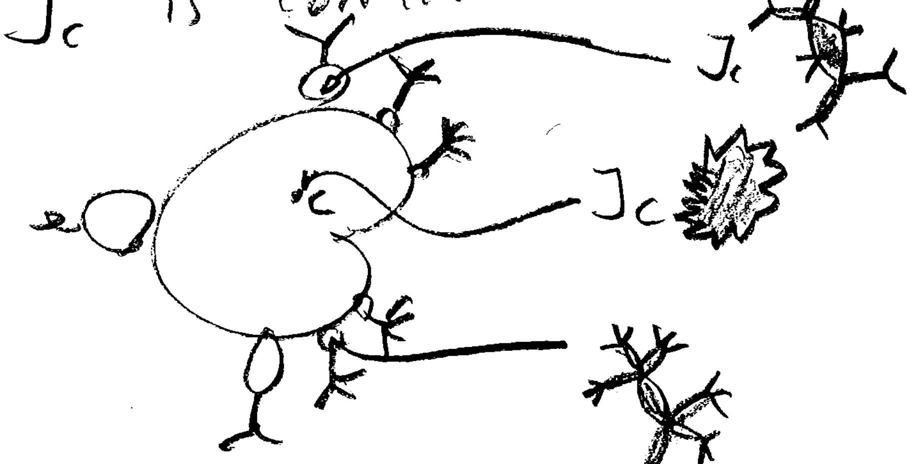
$$z_1 = z_0^2 + c$$
$$z_2 = z_1^2 + c$$
$$\vdots$$

does not run away to infinity

<u>Dichotomy</u> <u>theorem</u>: J_c is either connected (one piece) or is a dust

J_c is connected if and only if the iterates of $z_0 = 0$ do not diverge to ∞

The Mandelbrot set is the map of those c for which the Julia set J_c is connected.



PF 2 #5