1. Check explicitly that the Gauss-Bonnet theorem holds for circles in the unit sphere. That is, if $C$ is a circle of latitude and $D$ is the region of the sphere north of $C$, check that

$$\int_C kds + \text{Area}(D) = 2\pi.$$ 

What about the region $\overline{D}$ south of the same circle $C$?

2. The circles of intersection of the pseudosphere with planes orthogonal to its axis of rotation are called horocycles. Show that the geodesic curvature of any horocycle is 1.

3. Find an orthonormal frame field $X_1, X_2$ in the sphere minus the north and south poles. That is, $X_1$ and $X_2$ should be fields of unit tangent vectors with $X_1 \cdot X_2 = 0$.

Compute the vector fields $\nabla X_i X_j$ for each choice of $i, j$.

4. Suppose that a surface has a parameterization with domain $U$ in the upper half plane $\{(u, v) : v > 0\}$, and the first fundamental form is given by $E = G = 1/v^2, F = 0$.

Compute the Christoffel symbols for this surface. Determine whether the lines $\{u = \text{const.}\}$ or $\{v = \text{const.}\}$ are geodesics.