(1) Compute the exterior derivative of the following forms:
   (a) $\alpha = \sin x \, dx$
   (b) $\beta = \sin x \, dy$
   (c) $\omega = z^2 \, dx \wedge dy + x^2 \, dx \wedge dz$
(2) Let $\omega = y^2 \, dx + 2xy \, dy$ in the plane.
   (a) Compute $\int_\gamma \omega$, where $\gamma : [0, 2] \to \mathbb{R}^2$ is defined by
      
      \[
      \gamma(t) = \begin{cases} 
      (t, 0) & \text{if } t \in [0, 1] \\
      (1, t - 1) & \text{if } t \in [1, 2] 
      \end{cases}
      \]
   (b) Compute $\int_\beta \omega$, where $\beta : [0, 2] \to \mathbb{R}^2$ is defined by
      
      \[
      \beta(t) = (t/2, t/2).
      \]
   Compare with part (a).
(3) Prove that, if $\eta$ is any 1-form in $\mathbb{R}^n$, $\eta \wedge \eta = 0$. Is the same true for a 2-form? (Prove or give a counterexample).
(4) If $\gamma$ is a closed curve in $\mathbb{R}^2$ without self intersection, traversed counterclockwise, prove that $\int_\gamma x \, dy$ is the area enclosed by $\gamma$.
(5) Consider the following 1-form:
   $\omega = \frac{xdy - ydx}{x^2 + y^2}$
   (a) Compute the integral of $\omega$ around the circle of radius 2 (counterclockwise).
   (b) Show that $d\omega = 0$.
   (c) How do you reconcile (a) and (b) with Stokes’ theorem?
(6) Let $f : \mathbb{R}^3 \to \mathbb{R}$ and $h : \mathbb{R}^3 \to \mathbb{R}$ be smooth functions, and let $\omega = hdf$. Prove that $\omega \wedge d\omega = 0$. 