More homework due March 6

Problem 1

Recall that

$$\begin{aligned} GL(n,\mathbb{R}) &:= \{A \mid \det A \neq 0\} \\ O(n,\mathbb{R}) &:= \{A \mid AA^T = I\} \\ SO(n,\mathbb{R}) &:= \{A \in O(n,\mathbb{R}) \mid \det A = 1\} \\ GL(n,\mathbb{R})^+ &:= \{A \mid det(A) > 0\} \end{aligned}$$

1. Show that $O(n,\mathbb{R})$ is a deformation retract of $GL(n,\mathbb{R})$.
2. Show that $SO(n,\mathbb{R})$ is a deformation retract of $GL(n,\mathbb{R})^+$.
3. Show that $GL(n,\mathbb{R})^+$ is path connected. Conclude that $SO(n,\mathbb{R})$ is path connected.
4. Show that $GL(n,\mathbb{R})$ has precisely two path connected components.

Problem 2

A space has the homotopy extension property (HEP) with respect to a subspace $A \subset X$ if for every space Y and for all maps α, β that satisfy

$$\begin{aligned} \alpha : &\quad I \times A \to Y \\ \beta : &\quad \{0\} \times X \to Y \end{aligned}$$

with $\alpha(0,a) = \beta(0,a)$ for all $a \in A$, there is a map $F : I \times X \to Y$ extending α and β.

1. Suppose A is a closed subspace of X. Show that X has the HEP with respect to A if and only if $I \times A \cup \{0\} \times X$ is a retract of $I \times X$.
2. Show that B^n has the HEP with respect to S^{n-1}.