9 The Dynamics of Snow and Ice Masses

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9.1 Ice: Land, Sea and Air

On Earth today we enjoy a relatively comfortable climate, which is a fortunate consequence of the present extent of the global ice cover. Although more than two-thirds of the surface of Earth is covered by water, it is the water to ice conversion, and vice versa, that makes an important fraction of the globe habitable today. Hence, changes in the global scale dynamics of the ice cover capture scientific and public interest principally because of their role in global warming and ice-age events. It is in this sense that ice is the ultimate geomorphological fluid mechanic.

Field observations [1] and modeling studies [19] of past and "future" climates teach us that the ice cover is an extremely sensitive geophysical variable. Among other things, the eccentricity, obliquity and precession index of Earth's orbit, the optical depth of the atmosphere, and the storage of heat in the oceans underlie the present tropical-to-polar difference in mean surface temperature of approximately $50\,^o\,$C. Because water freezes near the middle of this range, the suggestion of advancing or retreating ice extent is not hard to grasp. Indeed, our contemporary polar oceans undergo dramatic seasonal variations in their sea-ice covers, amounting to approximately 18 million square kilometers in the Antarctic and 8 million square kilometers in the Arctic, where a perennial ice cover persists. The swift ice streams of West Antarctica are believed to modulate sea-level by influencing the storage of relatively slow inland (upstream) ice [2]. These contemporary observations give strength to the notion of rapid ice motion with consequences for all of Earth's inhabitants [3].

Most of what we study concerning the dynamics of the present ice cover involves our interest in understanding how, and how fast, circumstances might change. We study the past, as far back as 420,000 years, principally through the analysis of ice cores from the great ice sheets [1], for they trap in their polycrystalline matrix particulate and chemical clues concerning the history of the state of the Earth's past environments. Deriving a truly quantitative understanding of these environments constitutes a challenging inverse problem, for a host of post-depositional dynamical processes can act to redistribute climate proxies.

An ice sheet is maintained by the deposition of snow on its surface. The nucleation and growth of snow in the atmosphere occurs under chemical and dynamical conditions that mirror important aspects of climate. Our common experience tells us that a meter or so of snow can form a relatively loose aggregate of granular material, a fact in strong evidence during an avalanche. As

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snow accumulates on an ice sheet or glacier it is compressed into “firn”, which is less dense than ice and more dense than snow. In this layer, typically tens of meters thick, some atmospheric gases are displaced by the advection of seasonal meltwater or vapor transport through the connected network of air pockets that separate individual ice grains. Eventually, deeper down, the air pockets are sealed and the interfaces between the grains confine impurities. The persistence of unfrozen liquid separating these grain boundaries is a basic aspect of the phase behaviour of all polycrystalline material called *premelting* [24], [25]. But at the cold temperatures that persist near the surface of the polar ice sheets, such liquid is likely to be present only in small quantities. Hence, because of the extremely slow solid-state diffusion through single ice grains, the impurities are normally considered to be “frozen” in place. However, at higher temperatures and solute concentrations, premelted liquid at grain boundaries provides an alternative route for diffusive transport.

A proper accounting of the role of premelted liquid in the bulk diffusive properties of ice reveals that the interaction between compositional diffusion and the phase relationships determining the fraction of unfrozen liquid causes advection of the bulk-compositional signal towards warmer regions while maintaining its spatial integrity [17]. We can illustrate the basic effect of how the migration of such a climate signal evolves by modeling the diffusion of a single impurity such as H_2SO_4. What theory predicts is that, under conditions representative of those encountered in the Eemian interglacial ices of central Greenland, impurity fluctuations may be separated from ice of the same age by as much as half a meter, which is a distance comparable to the thickness of the apparent sudden-cooling events detected in Eemian ices from the GRIP core [17]. Moreover, when considering the premelting-enhanced diffusion of two species, we find that features of their evolution can mimic what has been ascribed to irreversible chemical reactions. The theory should help guide the analysis of existing and incipient deep ice cores. To the uninitiated it may seem paradoxical that a quantitiative understanding of the past global climate hinges on an understanding of the basic microscopics of ice, but it an unavoidable fact, and one that studies of future climates must also come to grips with.

The future is dealt with principally through the speculative viewing glass of prognostic global and regional models, which are initialized using various aspects of the present and past record. Although such models emerge out of the rostrum of geophysical fluid dynamics, they sacrifice the “rigor” of process studies to construct “realism” on the large scale. Large scale models incorporate a plethora of approaches which include various degrees of “realistic physics”, in order to study the sensitivity of a prediction to particularly well known feedbacks. Because of the sensitivity of the polar regions in model simulations and the recent changes in the Arctic climate [11], [18], air/sea/ice interactions are at the forefront of efforts to understand how the past climate has and can influence the future. During approximately the last decade a dramatic weakening of the central Arctic basin sea-level pressure, coupled with the European subarctic low pressure cell, ascribed to a positive phase of the North Atlantic Oscillation [6], has driven changes in polar atmospheric circulation [23]. These phenomena are part of a larger scale circulation pattern, called the Arctic Oscillation [22], with intraseasonal, interannual, and interdecadal time scales, interpreted as modulations in the strength of the polar vortex. Over the same time scales there have been large scale increases in surface atmospheric temperature and the temperature and salinity of the upper Arctic Ocean, the areal extent of sea ice has decreased in the Arctic and increased in the Antarctic, and large areas of permafrost have thawed [12].

These observed phenomena are difficult to model with entirely prognostic methods that do not restore forcing to deep-ocean climatology and/or observed atmospheric temperatures (e.g. [27]). The thermodynamic state of the atmosphere and the ocean are more readily observed than that of the sea ice that separates the two, and yet it is the change in the area of sea ice that drives ice-albedo feedback. Sea ice grows and melts in response to atmospheric and oceanic changes, it is redistributed by wind stress at its surface and it undergoes deformation in which ice of one thickness becomes ice of another [20]. The first thermodynamic models of sea ice developed for climate studies were one-dimensional and hence avoided the complication of deformation [10]. It was recognized then that the two-phase, two-component nature of the sea ice matrix, through its influence on the thermophysical properties of the layer, strongly influenced the agreement between model predictions and field observations. We now understand that a complete treatment of the thermodynamic properties of undeformed sea ice involves taking account of both diffusion and convection *within* the layer [26], and yet one of the principal simplifications of the original thermodynamic sea ice model [10] is to ascribe the same constant value of salinity to all ice thicknesses no matter what their particular history. Moreover, present day numerical models are not able to make an accurate accounting of the space-time variation in the distribution of sea ice thickness, but progress is being made [27], and when the observed trends and changes can be predicted reliably, the future may reveal itself. An enduring question concerns just which ostensibly small scale thermodynamic phenomena can be approximated to achieve a practical, computable scheme for making such predictions. Increasing computational power is not always the solution to the problem, for we are often in a position to make a "simplification" with the aim of enhancing computability, when in fact "simplification" is simply a synonym for ignorance of the underlying process. It is only years later, after that simplification becomes part of the fabric of the modeling, that the physics is rooted out.

There are other ways of predicting change. So called low-order models can focus our attention on the dominant balances in a system thereby providing insight on issues such as how sea ice responds to changes in poleward heat transport or ocean temperature [21]. Some variations, such as wind speed, temperature and atmospheric CO_2 content are predicted to be fast, whereas others, such as ice mass, are believed to respond more slowly, indicating a separation of time scales in the climate system [19]. Such a separation is again distinguished in studies of ice variations on daily, seasonal, decadal, millennial,...periods. The spatial extent of geophysical ice forms, and the characteristic length scales of processes controlling them, also provide natural divisions of ice research.

9.2 Ice Flow: As Clear as Mud

The essence of modern dynamical glaciology emerged out of basic considerations of the flow of ice as a problem in plasticity theory [14], [13], and we now understand that the thermal history of a parcel of glacier ice can be influenced by diffusion, advection and strain heating, and the temperature itself can modify chemical and isotopic signatures. Ice flows under its own weight and, because it builds up on, and creates, irregular landforms, understanding the dynamics of large snow and ice masses constitutes a formidable task [15]. Even along ice divides, which are the preferred drilling sites for ice cores because of their relatively simple flow pattern, reconstructing thermal and mechanical states is ultimately a computational undertaking [9].

In the present day, ice movements constitute one of the most important geomorphological sculptors in colder regions of the Earth, and in the past they were responsible for the landscape we presently observe in vast portions of the globe that are ice free. The considerations employed to great advantage in understanding sediment dynamics in rivers, lava and mud flow, dune formation, and the dynamics of ice masses, constitute a unifying theme for this volume, tying together these seemingly distinct flow processes that shape features of the Earth's surface.

It is the aspect ratio of the important features of these diverse flows that underlies their mathematical similarities. This is displayed in the generally shallow nature of glaciers and ice sheets, with depth to width/length ratios of about 10^{-3} [15], [7], [4]. A kind of minimal model of a glacier is depicted in Fig. 9.1. The glacier has thickness h, much smaller than its length and width, overlying

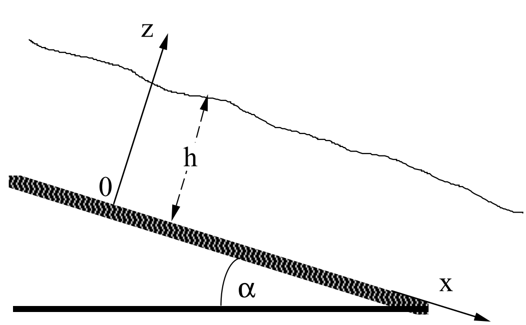

Fig. 9.1. Schematic of a parallel-sided slab [15]

a sticky inclined bed of slope α. In real glaciers, there is a difference between the surface and the bed slopes, but the essential behavior is not changed when this difference is small, except near margins, where the assumption tends to fail. The aspect ratio allows the neglect of longitudinal stresses, an assumption which breaks down in the vicinity of ice divides where the surface slope and basal shear stress vanish.

Traditionally, when employing these assumptions, dynamical glaciologists have referred to this treatment as the "parallel-sided slab" model (e.g. [15]), but as these phenomena drew other scientists into the field it became known as the *shallow-ice approximation* in analogy with the widely used *shallow-water equations* of geophysical fluid dynamics [16]. In the latter case, the essential notion is that the average fluid depth, which is the length scale characterizing the vertical scales of motion, is much smaller than the scale over which horizontal motions are important. Additionally, this approximation assumes that the density is constant in the layer and that the fluid is incompressible thereby decoupling the dynamics from the thermodynamics. The qualitatively similar approach of the shallow-ice approximation is used to study the surface profiles of ice sheets, to detect ice thickness changes and to reconstruct past ice sheets.

Ice sheets are rarely in steady state and hence mass conservation demands that

$$h_t + q_x = \mathcal{S} + \mathcal{B} \,, \tag{9.1}$$

where q is the ice flux and $\mathcal{S}$ and $\mathcal{B}$ are the mass per unit area per unit time being added or removed from the surface and bed of the glacier. Typically, the mass balance is dominated by the surface term. As in the case of *lubrication theory* of thin viscous flows, or in the evolution of dunes, the specific dynamics of the system resides in an understanding of the driving fluxes. Ice is often treated as a "viscous" fluid, but in fact it is a non-newtonian material with a power law rheology which for simple shear takes the form

$$\dot{\epsilon}_{xz} = A{\tau_{xz}}^3 \,, \tag{9.2}$$

wherein $\dot{\epsilon}_{xz}$ is the strain rate and τ_{xz} is the stress, A is an Arrhenius factor modelling the temperature dependent creep of ice (treated as a constant here), and the exponent 3 derives from Glen's experiment [5]. The ice flows under its own weight with a component of force parallel to the bed $\rho g h \sin\alpha$ which is balanced by the resisting stress at the bed. At a given depth in the ice the shear stress is $\tau_{xz} = \rho g(h - z)\sin\alpha$ and therefore, because $2\,\dot{\epsilon}_{xz} \equiv u_z + w_x$ and $w_x \approx 0$, we have

$$u(z) = u_s - \frac{A}{2}(\rho g h \sin\alpha)^3 (h - z)^4 \,, \tag{9.3}$$

where u_s is the surface velocity. Because the ice is frozen to the bed, the ice flux q can be written in terms of the depth averaged velocity $\bar{u}$ as

$$q = h\bar{u} = \frac{2A}{5}(\rho g h \sin\alpha)^3 h^5 \,, \tag{9.4}$$

and hence, neglecting transverse strain rate, the simplest shallow-ice model can be written

$$h_t + (h\bar{u})_x = \mathcal{S} + \mathcal{B} \,, \tag{9.5}$$

describing how the longitudinal strain rate varies with mass balance and thinning or thickening; a sudden increase in snow fall in the accumulation zone must result is a change in the ice thickness and a flux down glacier. Such simple models show that the dynamics of glaciers and ice sheets have the signature of climate variations encoded in them. Other examples wherein models of the form of (9.5) arise appear throughout the volume; resulting from the combination of conservation laws and shallow configurations.

9.3 Drumlins, Glaciers, Icebergs and Avalanches

In the section of the volume that follows, the authors discuss processes that span many of the space and time scales of active interest. Kolumban Hutter focuses on theories used to describe the dynamics of ice sheets and shelves such as the shallow-ice approximation. Such approaches develop the asymptotic limits that provide insight into the essential dynamics and often simplify the calculational efforts required to make long term predictions. When an ice sheet retreats it leaves geomorphological clues of its past existence in a region. One such clue takes the form of a long ridge, or oval-shaped hill, called a "drumlin" by glaciologists. Andrew Fowler presents a theory of drumlin formation in which these landforms emerge out of an instability coupling the interaction between the ice flow and the properties of the till below it. He also employs a shallow-ice methodology. Much of the hype associated with global warming is fueled by the periodic calving of gigantic icebergs from Antarctic ice shelves. An active area of present research concerns the armada of icebergs believed to have been calved from the Laurentide Ice Sheet during the last deglaciation [8]. The sinking of the Titanic focused our attention on the practical aspects of icebergs – their drift and deterioration – which is the topic of the chapter by Stuart Savage. A deeply aesthetic aspect of the hydrological cycle is the freezing of atmospheric water vapor to form snowflakes. The microscopics of this commonplace event have a plethora of macroscopic consequences [24] ranging from stratospheric processes to the ski slopes. Avalanche disasters in the Alps have a long history and yet a quantitative understanding is in its infancy. Much of the research, as described by Christophe Ancey, focuses on empirical correlations and modeling the dynamics like that of a debris flow. A great deal of damage can be mitigated, but in the long term a reliable predictive methodology is necessary.

As the present day snow and ice masses wax and wane, we think of the oscillations and their stability. Could an understanding of the qualitative mechanisms associated with the bifurcations of past climates, analogous to the patterns emerging out of convection in a layer of fluid, be sufficient to make progress in understanding the future? Nevertheless, the present is peppered with ice problems of pressing importance; problems suggested by studies of the past, the need to understand practical issues in the present and to make our best guess at what is ahead of us.

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References

1. R.B. Alley: Proc. Nat. Acad. Sci. **97**, 1331 (2000)
2. S. Anandakrishnan, D.D. Blankenship, R.B. Alley, P.L. Stoffa: Nature **394**, 62 (1998)
3. M.I. Budyko: *Climate and Life* (Academic Press, New York 1974)
4. A.C. Fowler: *Mathematical models in the applied sciences* (C. U. P., Cambridge 1997)
5. J.W. Glen: Proc. Roy. Soc. A **207**, 519 (1955)
6. J.W. Hurrell: Science **269**, 676 (1995)
7. K. Hutter: Theoretical Glaciology (D. Reidel, Dordrecht)
8. D.R. MacAyeal: Paleoceanography **8**, 775 (1993)
9. S.J. Marshall, K.M. Cuffey: Earth Planet. Sci. Lett. **179**, 73 (2000)
10. G.A. Maykut, N. Untersteiner: J. Geophys. Res. **76**, 1550 (1971)
11. M.G. McPhee, T.P. Stanton, J.H. Morison, D.G. Martinson: Geophys. Res. Lett. **25**, 1720 (1998)
12. J.H. Morison, K. Aagaard, M. Steele: Arctic **53**, 359 (2000)
13. J.F. Nye: Proc. Roy. Soc. Lond. **207**, 554 (1951)
14. E. Orowon: J. Glaciol. **1**, 231 (1949)
15. W.S.B. Paterson: *The Physics of Glaciers*, 3rd ed. (Pergamon, Oxford 1994)
16. J. Pedlosky: *Geophysical Fluid Dynamics*, 2nd ed. (Springer, New York 1987)
17. A.W. Rempel, E.D. Waddington, J.S. Wettlaufer, M.G. Worster: Nature **411**, 568 (2001)
18. D.A. Rothrock, Y. Yu, G.A. Maykut: Geophys. Res. Lett. **26**, 3469 (1999)
19. B. Saltzman, H. Hu, R.J. Oglesby: Dyn. Atmos. & Oceans **27**, 619 (1998)
20. A.S. Thorndike: J. Geophys. Res. **97**, 12601 (1992)
21. A.S. Thorndike: in *Ice Physics and the Natural Environment* NATO ASI, Series 1, Vol. 56 (ed. by J.S. Wettlaufer, J.G. Dash, N. Untersteiner) 169–184 (Springer–Verlag, Berlin 1999)
22. D.W.J. Thompson, J.M. Wallace: Geophys. Res. Lett. **25**, 1297 (1998)
23. J.E. Walsh, W.L. Chapman, T.L. Shy: J. Climate **9**, 480 (1996)
24. J.S. Wettlaufer: Phil. Trans. Roy. Soc. A **357**, 3403 (1999)
25. J.S. Wettlaufer, J.G. Dash: Sci. American **282**, 56 (2000)
26. J.S. Wettlaufer, M.G. Worster, H.E. Huppert: J. Geophys. Res. **105**, 1123 (2000)
27. J. Zhang, D.A. Rothrock, M. Steele: J. Climate **13**, 3099 (2000)