The (mis)Behavior of Markets

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A Fractal View of Risk, Ruin, and Reward

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PRELUDE

by Richard L. Hudson

Introducing a Maverick in Science

INDEPENDENCE IS A GREAT VIRTUE. To illustrate that, Benoit Mandelbrot relates how, during the German occupation of France in World War II, his father escaped death. One day, a band of Resistance fighters attacked the prison camp where he was being held. They disarmed the guards and told the inmates to flee before the main German force struck back. So the surprised and disoriented prisoners set off towards nearby Limoges, en masse and on the high road. After half a kilometer, Mandelbrot *père* decided this way was folly. So he set off by himself. He left the main group and the open road and broke off into the thick forest to walk back home alone. Shortly after, he heard a German Stuka dive-bomber strafe the main party of prisoners on the high road. He, alone in the forest, escaped harm. "It was," recalls the son, "the way my father behaved throughout his life. He was an independent man—and so am I."

Mandelbrot, a teenager during the war, is now famous. He got a Ph.D. in mathematical sciences in Paris, joined the influx of European scientists to America, and went on to a long career of sci-

entific discovery and acclaim. He invented a new branch of mathematics, fractal geometry; he applied it to dozens of improbably diverse fields; and he received numerous awards and much media attention. But his early wartime lessons in independence—he says he was *aguerri*, or war-hardened, by his experiences—made him always strike off in a direction different from the rest. He has thereby engendered much controversy, through which he persisted. He calls himself a maverick. By that, he means he has spent his life doing only what he felt right, sticking his nose where it was not always wanted, belonging to no particular scientific community.

"I have been a lone rider so often and for so long, that I'm not even bothered by it anymore," he says. Or, as a mathematically minded friend put it, he moves orthogonally—at right angles—to every fashion.

These facts about Mandelbrot's life are important to remember when meeting him, as in this book. What he says is not what they normally teach at the business schools at Harvard, London, Fontainebleau, or his own university, Yale. He has been premature, contrary to fashion, trouble-making, in virtually every field he has touched: statistical physics, cosmology, meteorology, hydrology, geomorphology, anatomy, taxonomy, neurology, linguistics, information technology, computer graphics, and, of course, mathematics. In economics he is especially controversial. His first appearance in the field, in the early 1960s, caused a storm. Paul H. Cootner, then a well-known economist at MIT, praised Mandelbrot's work as "the most revolutionary development in the theory of speculative prices" since the study began in 1900—and then he went on to criticize details of its contents and "Messianic tone." It has been like that ever since. The economics establishment knows him well, finds him intriguing, and has grudgingly adopted many of his ideas (though often without giving him full credit). That has made him one of the most important forces for change in the theory of finance. But the establishment also finds him bewildering.

So this book is an end-run, to a broader world and a broader

audience than can be found in the faculty lounges of Cambridge, Massachusetts, or Cambridge, England. What Mandelbrot has to say is important and immediately relevant to every professional in finance, every investor in the market, anyone who just wants to understand how money gets won and lost with such frightening rapidity.

From the start, Mandelbrot has approached the market as a scientist, both experimental and theoretical. Einstein famously said: "The grand aim of all science is to cover the greatest number of empirical facts by logical deduction from the smallest number of hypotheses or axioms." Such parsimony has been Mandelbrot's aim. To him, a stock exchange is a "black box," a system at once complex, variegated, and elusive, to be studied with conceptual and mathematical tools that build upon those of physics. Since he pioneered this approach in the 1960s, it has greatly evolved. It provides a scientific perspective on markets that is unlike any you will find in conventional books on investment, markets, and the economy.

Thus, reading this volume will not make you rich. But it will make you wiser—and may thereby save you from getting poorer.

I, CO-AUTHOR in this endeavor, first met Mandelbrot in 1997 when I was managing editor of the *Wall Street Journal*'s European edition. He showed up at our Brussels office with a mission to convince us that we should rethink how markets work. At first, he struck me as the "mad scientist" stereotype—flyaway white hair, very cerebral, intense convictions, a fondness for digression and disputation. But I and editor and publisher Phil Revzin, then my boss, listened politely and did what newspaper editors often do in such circumstances. What the heck? Print what he has to say, and see what happens.

A year later, when I was planning a business conference for the newspaper, I thought of inviting Mandelbrot to talk about risk. He stole the show. The conference-goers, among the best-known finan-

ciers, entrepreneurs, and CEOs in Europe—preeminent risk-takers, all—listened at first in bemusement. Not your usual conference speaker. Then they got sucked into his strange story. Some said he made more sense than their CFOs. Afterwards, in our audience-feedback survey, they rated him as best speaker of the day—tied only by Steve Ballmer, the Microsoft CEO.

As a scientist, Mandelbrot's fame rests on his founding of fractal geometry, and on his showing how it applies in many fields. A fractal, a term he coined from the Latin for "broken," is a geometric shape that can be broken into smaller parts, each a small-scale echo of the whole. The branches of a tree, the florets of a cauliflower, the bifurcations of a river—all are examples of natural fractals. The math eschews the smooth lines and planes of the Greek geometry we learn in school, but it has astonishingly broad applications wherever roughness is present—that is, nearly everywhere. Roughness is the central theme of his work. We have long had precise measurements and elaborate physical theories for such basic sensations as heat, sound, color, and motion. Until Mandelbrot, we never had a proper theory of the irregular, the rough—all the annoying imperfections that we normally try to ignore in life. Roughness is in the jagged edge of a metal fracture, the rugged coastline of Britain, the static on a phone line, the gusts of the wind—even the irregular charts of a stock index or exchange rate. As he puts it, "Roughness is the uncontrolled element in life."

Studying roughness, Mandelbrot found fractal order where others had only seen troublesome disorder. His manifesto, *The Fractal Geometry of Nature*, appeared in 1982 and became a scientific best-seller. Soon, T-shirts and posters of his most famous fractal creation, the bulbous but infinitely complicated Mandelbrot Set, were being made by the thousands. His ideas were also embraced immediately by another scientific movement, chaos theory. "Fractals" and "chaos" entered the popular vocabulary. In 1993, on receiving the prestigious Wolf Prize for Physics, Mandelbrot was cited for "having changed our view of nature."

MANDELBROT'S LIFE story has been a tale of roughness, irregularity,. and what he calls "wild" chance. He was born in Warsaw in 1924, and tutored privately by an uncle who despised rote learning; to this day, Mandelbrot says, the alphabet and times tables trouble him mildly. Instead, he spent most of his time playing chess, reading maps, and learning how to open his mind to the world around him.

His harsh education in war came soon enough. Unusually attentive to the footsteps of approaching trouble, the Jewish family moved in 1936 to Paris, where another uncle, Szolem Mandelbrojt (spellings differ in so wandering a family), had settled earlier as a mathematics professor. The war came, and young Mandelbrot was sent to a small town in the French countryside, at different times caring for horses or mending tools. An overcoat nearly undid him. His father had bought him a woolen coat in an orange, pseudo-Scotch plaid: It was hideous by anybody's standards, but warm and welcome in wartime. One day, the police stopped him and his younger brother. A tall man wearing just such an overcoat had been spotted earlier, fleeing the scene of a French Resistance attack on German headquarters. "That's him," a collaborator pointed. A case of mistaken identity. Mandelbrot was released, but took no chances: An opportunity arose, and he slipped out of town.

Mandelbrot's moment of self-discovery as a mathematician came in Lyon in 1944, where benefactors hid him in—appropriately—a school. He had a fake ID card and touched-up ration coupons. The staff asked no questions; theirs was, he recalls, "a passive kind of *résistance*." In the first week, he sat uncomprehending before the meaningless words and numbers on the blackboard. Then the professor embarked on a lengthy algebraic journey. Mandelbrot's hand shot up. "Sir, you don't need to make any calculations. The answer is obvious." He described a geometrical approach that yielded a fast, simple solution. Where others would have used a formula, he saw a picture. The teacher, skeptical at first, checked: Correct. And

Mandelbrot kept doing the same thing, in problem after problem, in class after class. As he relates it:

> It happened so fast I was not conscious of it. I would say to myself: This construction is ugly, let's make it nicer. Let's make it symmetric. Let's project it. Let's embed it. And all that, I could see in perfect 3-D vision. Lines, planes, complicated shapes.

Ever since, pictures have been his special aids to inspiration and communication. Some of his most important insights came, not from elaborate mathematical reasoning, but from a flash recognition of kinship between disparate images—the strange resemblance between diagrams concerning income distribution and cotton prices, between a graph of wind energy and of a financial chart. The creative essence of fractal geometry is to combine the formal and the visual. The ready intuition of fractal pictures has, today, made the subject a college course at Yale and other universities, and a popular addition to many high school math courses. But among "pure" mathematicians, Mandelbrot's approach was initially criticized. Not rigorous, they chided; the eye can mislead. But, Mandelbrot rejoins, observation often led him to conjectures that have stimulated and challenged the most skilled mathematicians; many of these problems remain unsolved. In any event, when science was young, he says, pictures were essential; think of the anatomical drawings of Vesalius, the engineering sketches of Leonardo, or the optics diagrams of Newton. Only in the nineteenth century, when the great edifice of algebraic analysis was perfected, did pictures become suspect as, somehow, imprecise.

In an ever-more complex world, Mandelbrot argues, scientists need both tools: image as well as number, the geometric view as well as the analytic. The two should work together. Visual geometry is like an experienced doctor's savvy in reading a patient's complexion, charts, and X-rays. Precise analysis is like the medical test results—the raw numbers of blood pressure and chemistry. "A good doctor

looks at both, the pictures and the numbers. Science needs to work that way, too," he says.

Mandelbrot's career has taken a jagged path. In 1945, he dropped out of France's most prestigious school, the École Normale Supérieure, on the second day, to enroll at the less-exalted but more appropriate École Polytechnique. He proceeded to Caltech; then—after a Ph.D. in Paris—to MIT; then to the Institute for Advanced Study in Princeton, as the last post-doc to study with the great Hungarian-born mathematician, John von Neumann; then to Geneva and back to Paris for a time.

Atypically for a scientist in those days, Mandelbrot ended up working, not in a university lecture hall, but in an industrial laboratory, IBM Research, up the Hudson River from Manhattan. At that time IBM's bosses were drawing into that lab and its branches a number of brainy, unpredictable people, not doubting they would do something brilliant for the company. In all kinds of ways, it was a wise policy. Scientifically, it yielded five Nobel Prize winners. But it was abandoned in the 1990s, as the company struggled to survive. Mandelbrot's research for IBM included the patterns of errors in computer communication and applications of computer analysis—even, at one point, for the company's president an investigation of stock-price behavior. During the 1980s, his computer-drawn Mandelbrot Set became an oft-repeated demonstration and a test of the processing power of IBM's then-new personal computers. But Mandelbrot's scientific activities and reputation went far beyond the confines of the lab at Yorktown Heights.

FOR MANDELBROT, economics has been both inspiration and curse. His study of financial charts in the 1960s helped stimulate his subsequent fractal theories in the 1970s and 1980s. He taught economics for a year at Harvard; and his first major paper in the field in 1962 (expanded and revised in 1963 and the next few years) was a study of cotton prices. In it, he presented substantial evidence

against one of the fundamental assumptions of what became "modern" financial theory. At that time, the theory was beginning to be entrenched in university economics departments—and it would soon become orthodoxy on Wall Street. As Mandelbrot continued his fractal studies, he often returned to economics. Each time, he probed how markets work, how to develop a good economic model for them—and, ultimately, how to avoid loss in them.

Today, some of his ideas are accepted as orthodoxy. As the last chapter will show, they are incorporated into some of the most-sophisticated mathematical models with which banks and brokerage houses manage money, into the ways math Ph.D.'s price exotic options or measure portfolio risk from Wall Street to the City of London. For the sake of historical precision, a technical listing is in order here. Mandelbrot was the first to take seriously and study the so-called power-law distributions. His 1962 argument that prices vary far more than the standard model allows—that their distributions have "fat tails"—is now widely accepted by econometricians. (Scientific nomenclature is not always straightforward. The probability distribution behind this particular approach is variously called L-stable, stable Paretian, Lévy, or Lévy-Mandelbrot.) Also accepted is his argument that, by their very essence, prices can vary by leaps and bounds rather than in a continuous blur; and likewise, his 1965 argument that price changes today are dependent on changes in the long past

These are all facts of financial life that Mandelbrot established early on and insisted upon, even though they ran counter to the theology of finance that was becoming established at about the same time. He also did pioneering work in many now-well-trodden avenues of economics. From 1965 he was publishing on what he soon called fractional Brownian motion and on the underlying concept of fractional integration, which had recently become a widespread econometric technique. In 1972, he published a multifractal model that incorporates and extends long tails and long dependence. His papers from the 1960s are the pillars upon which rest a

branch of the dismal science called "econophysics." In 1966 he developed a mathematical model explaining how rational market mechanisms can generate price "bubbles." And finally, he built multifractals on his 1967 notion of a "subordinated" trading time, developed with H. M. Taylor, that has also passed into the toolkit of some financial modelers—though it, like some of his other theories, is often credited to later researchers.

Indeed, as a financial journalist previously unmired in disputes of academic priority, I would say Mandelbrot's batting average for correctly analyzing market behavior would accord him a place in the Economics Hall of Fame. That record, alone, should make this book worth reading.

But plenty of Mandelbrot's other ideas remain controversial in economics: for instance, his theories of "scaling," of multifractal analysis, and of long-term dependence—all at the core of this book. One reason was hinted at in Cootner's original review. Before resuming his sharp-tongued critique, the MIT economist summarized the significance of what Mandelbrot had, at that early date, only begun to say:

> Mandelbrot, like Prime Minister Churchill before him, promises us not utopia but blood, sweat, toil and tears. If he is right, almost all of our statistical tools are obsolete—least squares, spectral analysis, workable maximum-likelihood solutions, all our established sample theory, closed distributions. Almost without exception, past econometric work is meaningless.

In 2004, in his eightieth year, Mandelbrot continues making trouble. He works the same full schedule—including weekends—as he always has. He continues publishing new research papers and books, lecturing at Yale, and traveling the world of scientific conferences to advance his views. Why not? After all, as he points out, Racine's most enduring play, *Athalie*; Verdi's greatest opera, *Falstaff*;

Wagner's *Ring Cycle*—all were written in the twilight of life, when the artist, after years of experience and experimentation, was at the height of his powers.

This book, too, is somewhat of an operatic performance—an interplay of voices, drama, and scenery. Throughout the main body of the book, the "I" voice is that of Mandelbrot, the ideas are his, and it is the drama of their discovery that motivates much of the text. The scenery is extensive and elaborate: Pictures, charts, and diagrams are key to understanding. And like the best operas, this book is written to be both engaging and popular. As the Notes and Bibliography suggest, a wealth of solid science and mathematics underpin our assertions—and the curious scientist or economist is welcome to consult those sources. All readers, of whatever background, are invited to visit the online addenda, at www.XXX.com/XXXX. It descends partly from a truly extraordinary Web site created by Mandelbrot's Yale colleague, Professor Michael Frame, for their popular undergraduate course on fractals for non-science majors, Math 190.

Today, Mandelbrot's message is more timely than ever, after a turbulent decade of bull markets, currency crises, bear markets, and the repeated building and bursting of asset bubbles. Financial markets are very risky places. And hitherto our understanding of them has been laden by the elaborate mathematics of orthodox financial theory—with many misguided assumptions, mis-applied equations, and misleading conclusions. Financial markets are complicated, but they need not be made overly so. To repeat: The aim of science is parsimony. The goal of this book is simplicity.