University of British Columbia
Student Name:

Math 101 Quiz 3

Section 203 - Winter 2014
Instructor: Jose Gonzalez
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Instructions: Please solve the exercises below. To get any credit you must provide a complete explanation.

(1) (*5 points*) Determine if the following series converges or diverges: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1}$

(2) (*5 points*) Determine the exact value of the series: $\displaystyle\sum_{n=0}^{\infty} \frac{2 \cdot 3^n}{5 \cdot 5^n}$

(3) (*5 points*) Determine if the following series converges or diverges: $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \cdot (\ln(n))^2}$

Solution (1)

We have that $0 < 2n-1 \leq 2n$ for all $n \geq 1$, so $0 \leq \frac{1}{2n} \leq \frac{1}{2n-1}$ for all $n \geq 1$. The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2} \cdot \sum_{n=1}^{\infty} \frac{1}{n}$ diverges, because $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ is a p-series with $p = 1$. Then by **comparison test** the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1}$ diverges as well.

Solution (2)

The given series is a **geometric series** with ratio $r = \frac{3}{5}$, and since $|r| = \frac{3}{5} < 1$ it converges. Its initial value is $a = \frac{2}{5}$. Then the value of the series is $\displaystyle\sum_{n=0}^{\infty} \frac{2 \cdot 3^n}{5 \cdot 5^n} = \frac{a}{1-r} = \frac{2/5}{1-3/5} = 1.$

Solution (3)

Let $f(x) = \frac{1}{x \cdot (\ln(x))^2}$. Since $x \cdot (\ln(x))^2$ is positive, continuous and increasing for $x > 1$, then $f(x)$ is positive, continuous and decreasing for $x > 1$, so we can apply **integral test**. We compute the corresponding improper integral:

$$\int_2^{\infty} f(x)\,dx = \int_2^{\infty} \frac{1}{x \cdot (\ln(x))^2}\,dx = \lim_{t\to\infty} \int_2^{t} \frac{1}{x \cdot (\ln(x))^2}\,dx = \lim_{t\to\infty} -\frac{1}{\ln(x)}\bigg|_2^t = \lim_{t\to\infty} \left(-\frac{1}{\ln(t)} + \frac{1}{\ln(2)}\right) = \frac{1}{\ln(2)}.$$

The improper integral converges, so by the integral test the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \cdot (\ln(n))^2}$ converges as well.