

INSTRUCTIONS (Read carefully): Please answer the **two** questions below. **Only the front side of the page is marked** (so you can use the back side find the solution and to check your work, but the full solution with the procedure must be at the front page to get marks). To get credit you need to **show all your work**.

Consider the vectors $w = \begin{bmatrix} 17 \\ -4 \\ -3 \\ -1 \\ -2 \end{bmatrix}$, and $v_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 3 \\ -2 \\ 4 \\ 5 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 5 \\ 1 \\ -4 \\ 3 \\ 6 \end{bmatrix}$, in the space \mathbb{R}^5 .

(1) Suppose that you want to determine if the vectors v_1 , v_2 and v_3 are linearly independent. Write a linear system of equations **in its matrix form** that would allow you to solve this problem. You do not need to solve the equations.

Solution The vectors v_1 , v_2 and v_3 are linearly independent when the following homogeneous linear system has the zero vector as its unique solution:

$$\begin{bmatrix} 1 & -1 & 5 \\ 2 & 3 & 1 \\ -1 & -2 & -4 \\ 3 & 4 & 3 \\ 2 & 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(2) Suppose that you want to determine if the vector w is a linear combination of the vectors v_1 , v_2 and v_3 . Write a linear system of equations **in its matrix form** that would allow you to solve this problem. You do not need to solve the equations.

Solution The vector w is a linear combination for the vectors v_1 , v_2 and v_3 when the following linear system has solutions:

$$\begin{bmatrix} 1 & -1 & 5 \\ 2 & 3 & 1 \\ -1 & -2 & -4 \\ 3 & 4 & 3 \\ 2 & 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 17 \\ -4 \\ -3 \\ -1 \\ -2 \end{bmatrix}$$