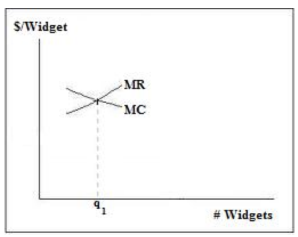
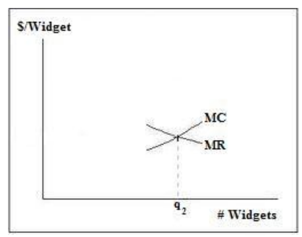
Math 115 Quiz 9

Section 4.5: Applications to marginality

(1) A friend of yours recently graduated from Harvard with a degree in economics and accepted a high-paying job as an analyst for Acme Widgets. He is currently trying to determine the number of widgets the company should be producing to maximize profit. He remembers something about needing to set marginal revenue equal to marginal cost, but his curves intersect in two different places, q_1 and q_2, so he can't decide which output level to choose. He knows that you, as a soon-to-be graduate of Michigan Calculus, have a far greater understanding of the situation than he does, so with his job (and indeed the future of Acme Widgets) on the line, he comes to you for advice. He shows you the following close-ups of his marginal revenue and marginal cost curves:





What advice do you give him about producing q_1 widgets? What about q_2? Are there any other possibilities he should consider?

Answer: (**Corrected!**) Remember that the profit is equal to $\pi = R - C$ and its derivative is equal to $\pi' = R' - C' = MR - MC$. Then both q_1 and q_2 are critical points of the profit function. We need to test these critical points in order to find information for our friend. We can tell him that since at q_1 the function $\pi' = R' - C' = MR - MC$ passes from negative to positive, then q_1 is a local minimum of the profit. In the same way, at q_2 the function $\pi' = R' - C' = MR - MC$ passes from positive to negative, then q_2 is a local maximum of the profit. In general, to maximize profit our friend needs to look at the values of the profit at the critical points and he also needs to consider the extreme cases, this is, he also needs to consider what happens when producing zero widgets and what happens in the limit when they produce very large quantities of widgets. **Note:** With the given information we can not tell what number of widgets maximizes profit.

Section 5.1: How do we measure distance traveled?

(2) The velocities $v(t)$ in kilometers per minute delivered by a certain bicycle with broken breaks, gears, chain, reflective lights, and kickstand (which in addition makes it difficult to pedal and almost impossible to turn), at different times t in minutes are given in the following table:

t	0	5	10	15	20
v(t)	0.08	0.14	0.16	0.10	0.12

The table stops at $t = 20$ since no one can really pedal this bike longer than 20 minutes. The irresponsible owner of this bike heads to his home located 3 kilometers away at time $t = 0$.

(a) What does $\int_0^{20} v(t)\,dt$ represent for the irresponsible owner of this bike?

Answer: It represents how far this hypothetical irresponsible person will travel in the twenty minutes he is able to ride his bike.

(b) Calculate the left-hand sum and the right-hand sum for the definite integral using the table.

$$\text{Left-hand sum} = 5 \cdot 0.08 + 5 \cdot 0.14 + 5 \cdot 0.16 + 5 \cdot 0.10 = 2.4km$$

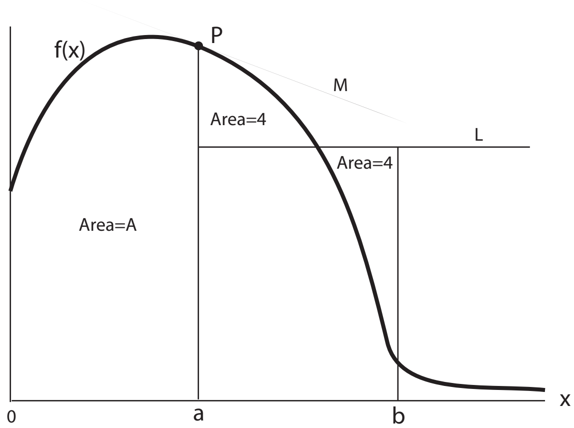
$$\text{Right-hand sum} = 5 \cdot 0.14 + 5 \cdot 0.16 + 5 \cdot 0.10 + 5 \cdot 0.12 = 2.6km$$

(c) If you knew that both of your estimations in **(b)** are within half a kilometer of the value in **(a)**, do you have any sad news for him before he goes home (besides the fact that his bike is broken)?.

Answer: The sad news is that he will make it to at most $2.4 + 0.5 = 2.9$ kilometers from his starting point, so his bike will not take him all the way home, and he will have to walk at the end.

Section 5.2: The definite integral

(3) Below you will write expressions for each of various quantities indicated on the graph of $f(x)$. Your expressions may involve integrals or derivatives. For example, if asked for the "x-coordinate of the point P," you would write "a".



a) The height (above the x-axis) of the point P.

Answer: $f(a)$

b) The slope of the line M

Answer: $f'(a)$

c) The size of the area A.

Answer: $\int_0^a f(x)\,dx$

d) The height of the line L

Answer: $\frac{1}{b-a} \cdot \int_a^b f(x)\,dx$