

Using the basic properties and the short-cuts for differentiation, find the derivative of the following function. You do not need to simplify your answers.

$$f(x) = \cos( (\ln(3x^7 + 2x))^4 \cdot \sin(4\sqrt[3]{x}) )$$

**Hints:** (i) Before writing your answer you might want to compute the derivatives of the following functions:

$$g(x) = (\ln(3x^7 + 2x))^4$$

$$h(x) = \sin(4\sqrt[3]{x})$$

(ii) Also, notice that:  $f(x) = \cos( g(x) \cdot h(x) )$ .

## Solution

$$g'(x) = 4 \cdot (\ln(3x^7 + 2x))^3 \cdot \frac{1}{3x^7 + 2x} \cdot (21x^6 + 2)$$

$$h'(x) = \cos(4\sqrt[3]{x}) \cdot \ln 4 \cdot 4\sqrt[3]{x} \cdot \frac{1}{3} \cdot x^{-2/3}$$

Since  $f(x) = \cos( g(x) \cdot h(x) )$ , using the chain rule and the product rule we obtain:

$$f'(x) = -\sin( g(x) \cdot h(x) ) \cdot (g'(x) \cdot h(x) + g(x) \cdot h'(x)),$$

and then we replace to obtain the answer:

$$f'(x) = -\sin( (\ln(3x^7 + 2x))^4 \cdot \sin(4\sqrt[3]{x}) ) \cdot \left[ \frac{4 \cdot (\ln(3x^7 + 2x))^3 \cdot (21x^6 + 2)}{3x^7 + 2x} \cdot \sin(4\sqrt[3]{x}) + (\ln(3x^7 + 2x))^4 \cdot \cos(4\sqrt[3]{x}) \cdot \frac{\ln 4 \cdot 4\sqrt[3]{x} \cdot x^{-2/3}}{3} \right].$$