

(1) Let $q(x) = (1 + e^{-x})^{-1}$

a) Calculate the derivative $q'(x)$. No need to simplify.

Solution

$$q'(x) = (-1)(1 + e^{-x})^{-2}(-e^{-x}) = \frac{1}{e^x(1 + e^{-x})^2}.$$

b) Imagine inserting your answer from part a) into the integral below. What does the fundamental theorem of calculus tell you about the integral? You need not calculate a value.

$$\int_{-1000}^{1000} (\text{answer from part a}) dx =$$

Solution

Using the Fundamental Theorem of Calculus, we obtain

$$\int_{-1000}^{1000} \frac{1}{e^x(1 + e^{-x})^2} dx = \int_{-1000}^{1000} q'(x) dx = q(1000) - q(-1000) = (1 + e^{-1000})^{-1} - (1 + e^{1000})^{-1}.$$

(2) $\int_3^9 (5 - 2f(x)) dx = 22$. Find $\int_3^9 f(x) dx$.

Solution

We have

$$\int_3^9 (5 - 2f(x)) dx = \int_3^9 5 dx - 2 \int_3^9 f(x) dx = 30 - 2 \int_3^9 f(x) dx.$$

Then $30 - 2 \int_3^9 f(x) dx = 22$. We solve for $\int_3^9 f(x) dx$, and we obtain

$$\int_3^9 f(x) dx = 4.$$