University of British Columbia
Student Name:

Math 184 Quiz 4

Team 922 - Summer 2013
Instructor: Jose Gonzalez
Date: June 17 / 2013

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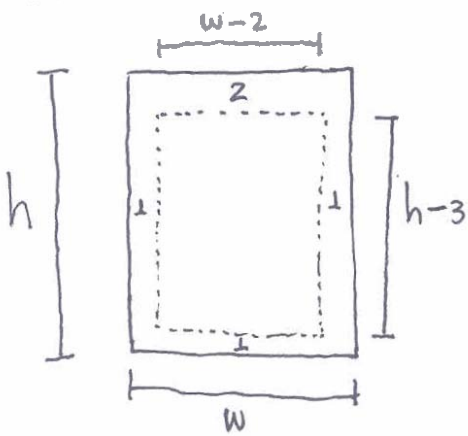
(1) (12pts) A rectangular poster is to have a total area of 600 in^2 (counting its rectangular printed area and its margins), with 1-inch margins at the bottom and sides and a 2-inch margin at the top. We want to find the dimensions that would give the largest printed area.

a) (8pts) In this part you are required to set up this problem, but not to solve it: Let h be the total height of the poster in inches. In the first box below express the area A of the printed section as a function of the total height h. In the second box below write the interval in which we want to maximize this function $A = A(h)$, in other words, write the possible values of h given the practical restrictions imposed by the problem. Write your answers in the boxes but show all your work.



Target function:

$$A = (h-3)(w-2).$$

$$wh = 600 \Rightarrow w = \frac{600}{h}$$

$$\boxed{A = (h-3)\left(\frac{600}{h} - 2\right)}$$

Interval:

$$h \geq 3$$

$$\frac{600}{h} = w \geq 2 \Rightarrow 300 \geq h.$$

$$\boxed{3 \leq h \leq 300}$$

Function A in terms of only h: $A = (h-3)\left(\frac{600}{h} - 2\right)$	Interval for h: $[a,b] = [3, 300]$

b) (4pts) For the function $A(h)$ that one finds above, one gets $A'(h) = \frac{1800}{h^2} - 2$. In this way $A'(h)$ always exists in the desired interval, and $A'(h)$ is zero in the interval only when $h = 30$. What is the total height h that gives the maximum printable area? Justify.

$A(h)$ continuous on $[3, 300]$. $\Rightarrow$ Enough to compare crit. pts & endpts.

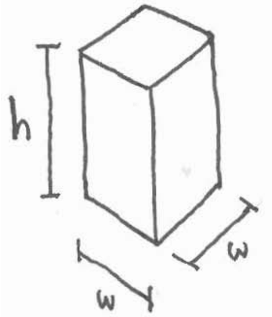
end pts $\begin{cases} A(3) = 0 \\ A(300) = 0 \end{cases}$

crit pts. $\{ A(30) = 486.$

$\Rightarrow h = 30$ inches gives the maximum printable area.

(2) (12pts) You want to design a milk carton box with a square base of side length w in cm, and with a height h in cm which holds 2000 cm^3 of milk. The sides of the box cost 1 $cent/cm^2$ and the top and bottom cost 2 $cent/cm^2$. The goal is to find the dimensions of the box that minimize the total cost of materials used.

a) (8pts) In this part you are required to set up this problem, but not to solve it: In the first box below express total the cost C in cents of a milk carton box as a function of the side of the square base w. In the second box below write the interval in which we want to minimize this function $C = C(w)$, in other words, write the possible values of w given the practical restrictions imposed by the problem. Write your answers in the boxes but show all your work.



Target function:

$C = 2 \cdot (w^2 + w^2) + 1 \cdot (wh + wh + wh + wh)$

$C = 4w^2 + 4wh$

$w^2 h = 2000 \Rightarrow h = \frac{2000}{w^2}$

$C = 4w^2 + 4w \cdot \frac{2000}{w^2}$

$$\boxed{C = 4w^2 + \frac{8000}{w}}$$

Interval:

$(0, \infty)$

Function C in terms of only w: $C = 4w^2 + \frac{8000}{w}$	Interval for w: $(a, b) = (0, \infty)$

b) (4pts) For the function $C(w)$ that one finds above, one gets $C'(w) = 8w - \frac{8000}{w^2}$. In this way $C'(w)$ always exists in the desired interval, and $C'(w)$ is zero in the interval only when $w = 10$. What is the side of the base w that gives the desired minimum total cost? Justify.

$C''(w) = 8 + \frac{16000}{w^3} > 0$ for all w in $(0, \infty)$.

Then the continuous function $C(w)$ is concave up in $(0, \infty)$, $w = 10$ is a local minimum by the second derivative test, and it is the only local extremum of $C(w)$ on $(0, \infty)$.

$\Rightarrow W = 10$cm is the absolute minimum, and gives the minimum total cost for the box.

(3) (6pts) Let $f(x)$ be a function that is continuous on the interval $[-1, 4]$. The function $f(x)$ is twice differentiable except at $x = 1$, and $f(x)$ and its derivatives have the properties indicated in the table below, where DNE indicates that the derivatives of $f(x)$ do not exist at $x = 1$.

x	-1	$-1<x<0$	0	$0<x<1$	1	$1<x<2$	2	$2<x<4$	4
$f(x)$	1	positive	0	negative	-1	negative	0	positive	3
$f'(x)$	-6	negative	0	negative	DNE	positive	0	positive	8
$f''(x)$	3	positive	0	negative	DNE	negative	0	positive	4

On the axes provided, sketch the graph of a function that has all the characteristics of $f(x)$