University of Michigan
Student Name:

Math 105 Quiz 4

Section 101 - Spring 2011
Instructor: Jose Gonzalez
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(1) (10 pts.) Solve the two exercises presented below:

(a) (5 pts.) Three tables are given below. The function $h(x)$ is the composition of the functions $g(x)$ and $f(x)$. Specifically:

$$h(x) = g\left(f(x)\right)$$

Complete the three tables. Assume that different values of x lead to different values of $g(x)$.

x	-2	-1	0	1	2
$f(x)$	4			5	1

x	1	2	3	4	5
$g(x)$		1	2	0	-1

x	-2	-1	0	1	2
$h(x)$		1	2		-2

Solution: We need to find $f(-1)$, $f(0)$, $g(1)$, $h(-2)$ and $h(1)$. We can compute all of these using the information in the tables and the relation $h(x) = g\left(f(x)\right)$. We have $g(f(-1)) = h(-1) = 1 = g(2)$, and then $f(-1) = 2$. Similarly, $g(f(0)) = h(0) = 2 = g(3)$, and then $f(0) = 3$. We have $g(1) = g(f(2)) = h(2) = -2$. We also have, $h(-2) = g(f(-2)) = g(4) = 0$ and similarly $h(1) = g(f(1)) = g(5) = -1$.

(b) (5 pts.) Showing all your, find the formula for the inverse of the function

$$k(x) = \frac{\sqrt{x} - 1}{\sqrt{x} + 1}.$$

Solution: Let $y = k(x)$, so $x = k^{-1}(y)$. We have:

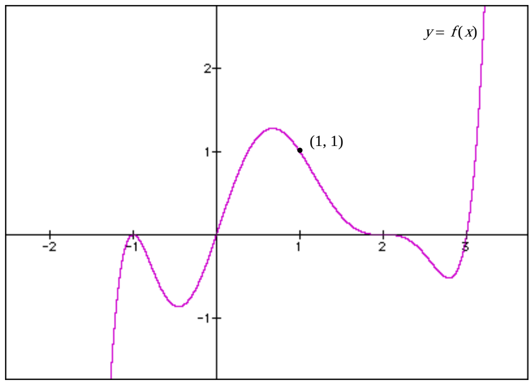
$$y = \frac{\sqrt{x} - 1}{\sqrt{x} + 1} \quad \Longrightarrow \quad (\sqrt{x} + 1)y = \sqrt{x} - 1 \quad \Longrightarrow$$

$$\sqrt{x}y + y = \sqrt{x} - 1 \quad \Longrightarrow \quad \sqrt{x}y - \sqrt{x} = -y - 1 \quad \Longrightarrow$$

$$\sqrt{x}(y - 1) = -y - 1 \quad \Longrightarrow \quad \sqrt{x} = \frac{-y - 1}{y - 1} \quad \Longrightarrow$$

$$x = \left(\frac{-y - 1}{y - 1}\right)^2 \quad \Longrightarrow \quad k^{-1}(y) = \left(\frac{-y - 1}{y - 1}\right)^2$$

(2) (10 pts.) Your ultimate goal in this question is to find an equation for the polynomial function $f(x)$ whose graph is shown below. The only letters that should appear in your final answer are $f(x)$ and x.



(a) (4 pts.) Find the ALL of the roots and some possible multiplicities of the function $f(x)$. Record your answers in the table shown below.

Root	Multiplicity

Answer: From the graph we see that the roots (a.k.a. the zeroes) are: $x = -1$, $x = 0$, $x = 2$ and $x = 3$, and that some possible multiplicities are 2, 1, 3 and 1, respectively. Note: The multiplicity of $x = -1$ should be even and the other three multiplicities should be odd, and also, by the shape of the graph we see that the multiplicity of the zero $x = 2$ should be at least 3.

(b) (6 pts.) Use your answers to Part (a) of this problem to find a formula for the polynomial function $f(x)$.

Answer: From (a) there is an equation for $f(x)$ that has the form

$$f(x) = a(x+1)^2 x(x-2)^3(x-3),$$

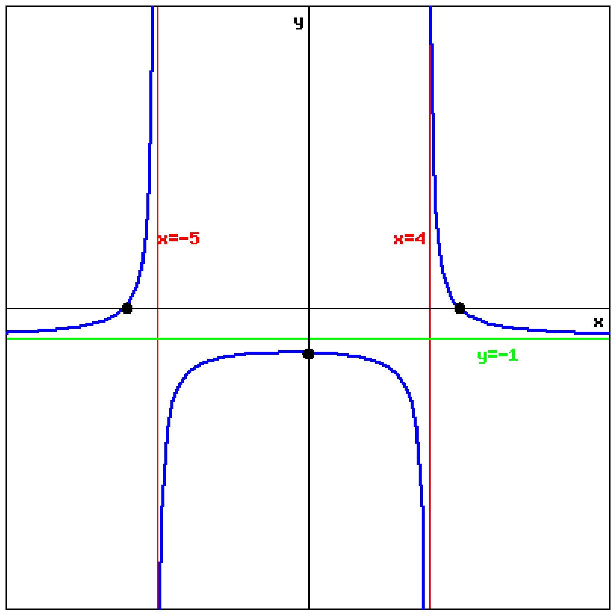
for some constant a. We use that the point $(1,1)$ is in the graph of $f(x)$ to find the constant a. We have:

$$1 = a(1+1)^2 1(1-2)^3(1-3) \Longrightarrow a = \frac{1}{8}$$

Then a possible equation for the polynomial function $f(x)$ is

$$f(x) = \frac{1}{8}(x+1)^2 x(x-2)^3(x-3)$$

(3) (10 pts.) Find a possible formula for the function $f(x)$ graphed below. The x-intercepts are marked with points located at $(5, 0)$ and $(-6, 0)$, while the y-intercept is marked with a point located at $(0, -\frac{30}{20})$.



Solution Since the graph has vertical asymptotes at x=4 and x=-5, let the denominator be $(x-4)(x+5)$. Since the graph has zeros at $x = 5$ and $x = -6$ let the numerator be $(x-5)(x+6)$. Since in the the long-run the rational function $f(x)$ tends toward $y = -1$ as x $\to \pm\infty$, the ratio of the leading terms should be -1.

So a possible formula is $y = f(x) = -\dfrac{(x-5)(x+6)}{(x-4)(x+5)}$. You can check that the y-intercept is y=-30/20, as it should be.